



Single and two-particle spectral functions across the disorder-driven superconductor- insulator transition

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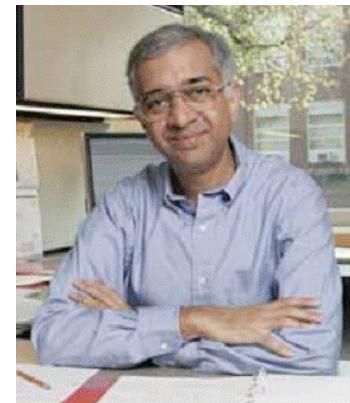
Karim
Bouadim



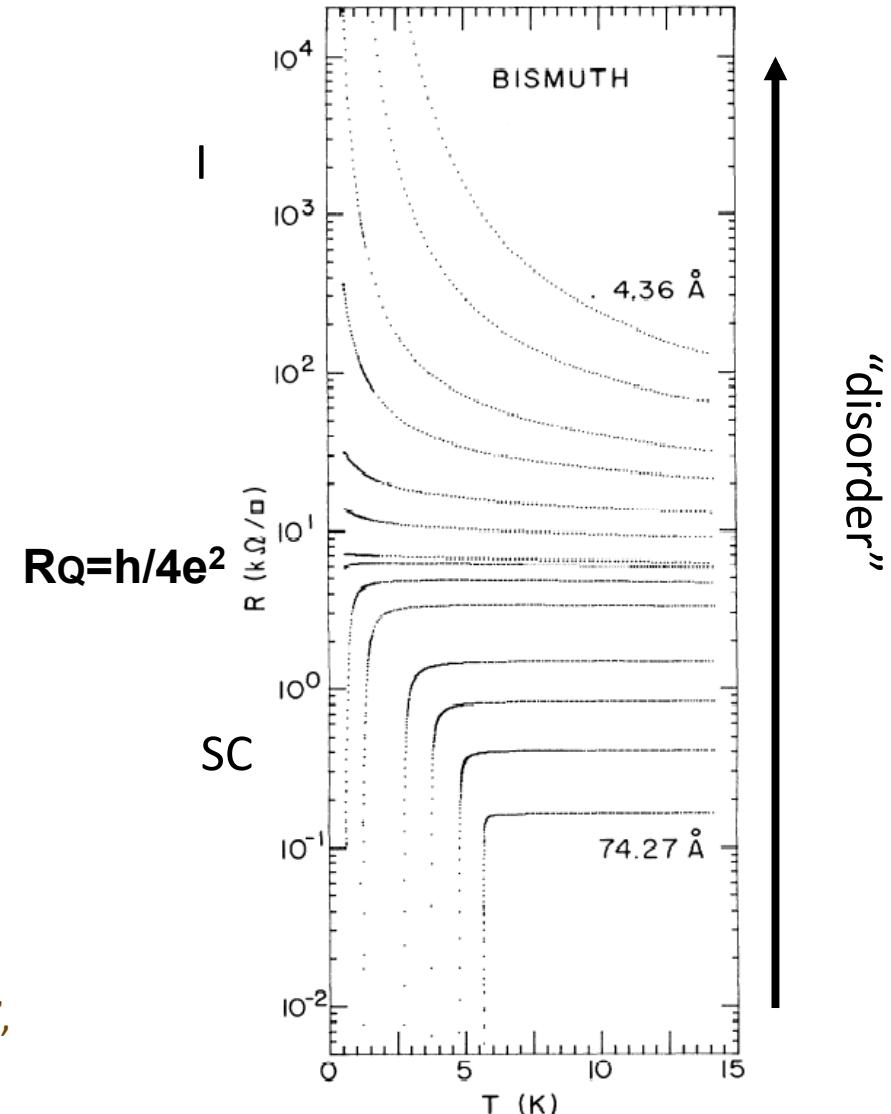
Yen Lee
Loh



Mohit
Randeria



Superconductor-Insulator Transition

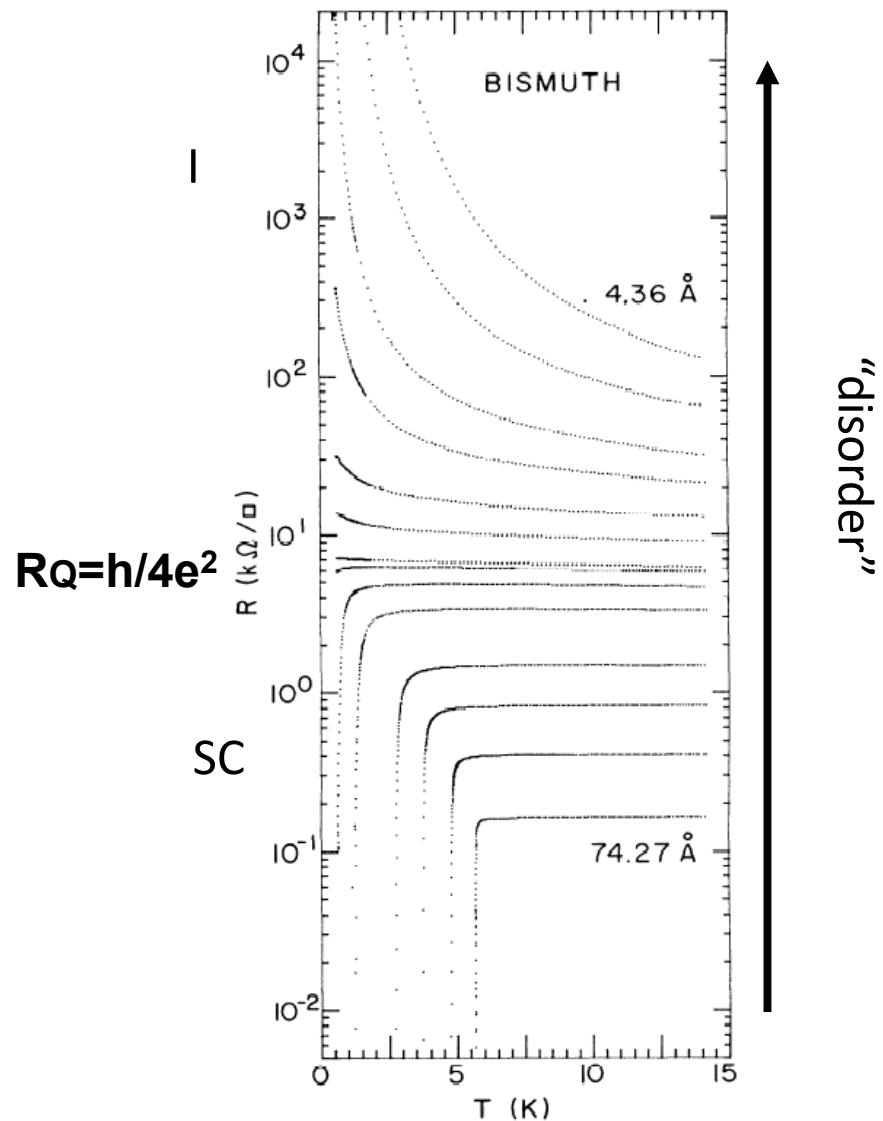
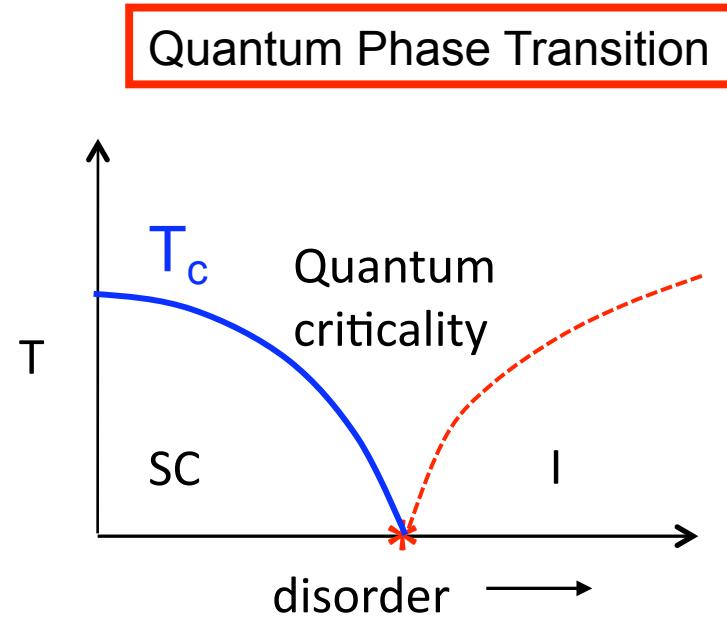


Also:

- Valles et.al. PRL 69, 3567 ('92)
- Hebard in "Strongly Correlated Electronic Systems", ed. Bedell et. al. ('94)
- Goldman and Markovic, Phys. Today 51, 39 (1998)
- Gantmakher and Dolgopolov arXiv:1004.3761

*Haviland et al. PRL 62 2180
(1989)*

Superconductor-Insulator Transition



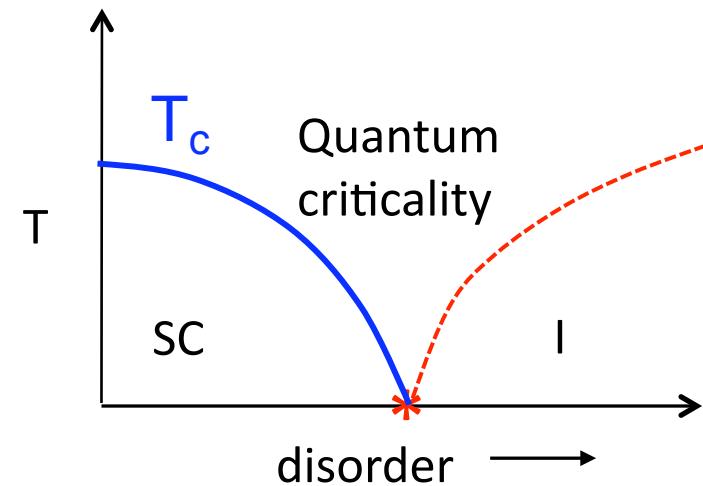
Haviland et al. PRL 62 2180
(1989)

Superconductor-Insulator Transition

BIG QUESTIONS....

Nature of

- *Normal state*
- *Insulator*
- *Crossover energy scale on insulating side*



Superconductor

(1) Electrons pair up

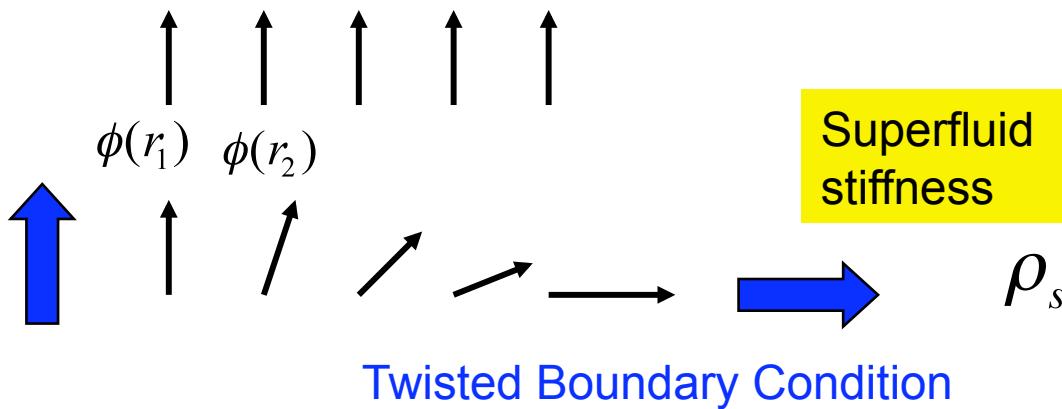
$$\Delta(r) = \langle c_{r\uparrow}^+ c_{r\downarrow}^+ \rangle = \Delta_0 e^{i\phi}$$

Egap

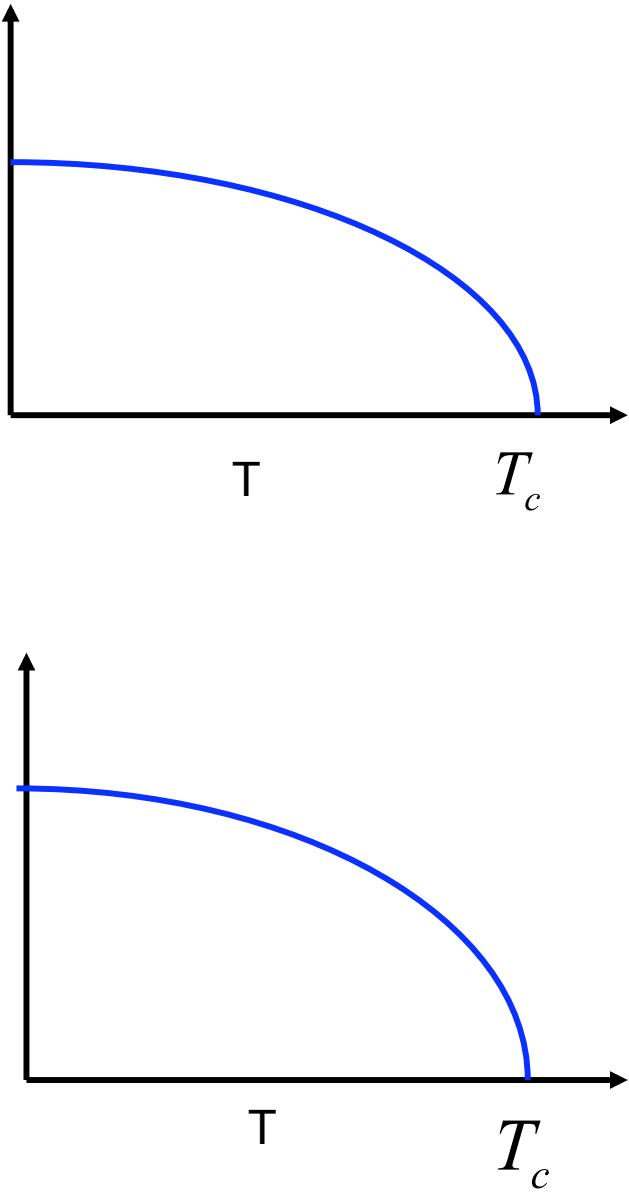
Δ_0

T T_c

(2) Long range phase coherence



$$\Delta E = \rho_s (\nabla \phi)^2$$



Homogeneous Films

Amplitude Driven

Granular Films

Phase Driven

Homogeneous Films

Amplitude Driven

Granular Films

Phase Driven

Are fermions important at the transition?

NO

YES



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Are fermions important at the transition?

NO

YES

Dirty Boson Models
universal properties at transition
Cooper pair \sim boson

MPA Fisher, Grinstein and Girvin
PRL 1990

Coulomb repulsion and disorder
 $\Rightarrow T_c$ degradation

Finkelshtein Physics B 1994;
JETP Let. 45, 46 (1987)

Previous work:

Pairing of exact eigenstates + uniform Pairing amplitude

Ma and Lee PRB 32, 5658 (1985)

Kotliar and Kapitulnik PRB 1986

Ramakrishnan Phys. Scripta 1989

SC not destroyed until the extreme disorder limit of site localization

Homogeneous Films

~~Amplitude Driven??~~

Granular Films

Phase Driven

Are fermions important at the transition?

NO

YES

Dirty Boson Models
universal properties at transition
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Coulomb repulsion and disorder
 $\Rightarrow T_c$ degradation

Finkelshtein Physics B 1994

Strong Amplitude (local gap) Fluctuations

But gap remains finite everywhere

Self generated granularity (SC puddles)

Transition is driven by quantum phase fluctuations

Outline:

- Model
- Calculational Methods
- Single and Two-particle Spectral Functions
- Phase Diagram

Model: Attractive Hubbard + disorder + field

$H =$ Kinetic energy

+

$$-t \sum_{\langle i,j \rangle \sigma} c_{i\sigma}^+ c_{j\sigma} + h.c.$$

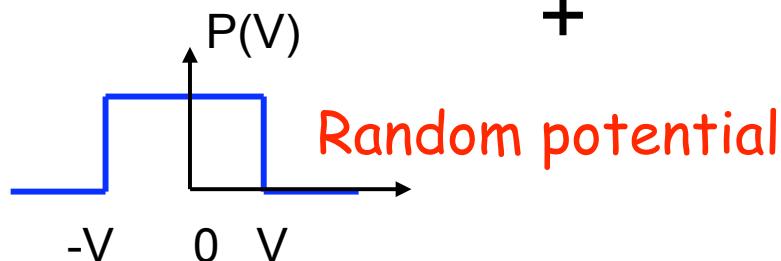
2D



Attraction

(U controls size of Cooper pairs)

$$-|U| \sum_i n_{i\uparrow} n_{i\downarrow}$$



+

$$+ \sum_i (V_i - \mu - h\sigma) n_i$$

Zeeman Field

$V=0$ s-wave SC

$|U|=0$ localization problem of non-interacting electrons

Methods

Bogoliubov-de Gennes-Hartree-Fock MFT

- Local expectation values
- Solve self consistently

$$n_\sigma(r) \equiv \langle c_{r\sigma}^+ c_{r\sigma} \rangle$$

$$\Delta(r) \equiv |U| \langle c_{r\uparrow}^+ c_{r\downarrow}^+ \rangle \equiv |U| F(r)$$

BdG keeps only amplitude fluctuations

Methods

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Local pairing amplitude

BdG keeps only amplitude fluctuations

Determinantal Quantum Monte Carlo: "exact"

No sign problem for any filling

Keeps both amplitude and phase fluctuations

Methods

Bogoliubov-de Gennes-Hartree-Fock MFT

- Local expectation values
- Solve self consistently

$$n_\sigma(r) \equiv \langle c_{r\sigma}^+ c_{r\sigma}^- \rangle$$
$$\Delta(r) \equiv |U| \langle c_{r\uparrow}^+ c_{r\downarrow}^+ \rangle \equiv |U| F(r)$$

BdG keeps only amplitude fluctuations

Determinantal Quantum Monte Carlo: "exact"

No sign problem for any filling

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Maximum entropy method for analytic continuation

$$N(\omega) = \frac{1}{N} \sum_k A(k, \omega) \quad G(k, \tau) = -\langle T c_{k\sigma}(\tau) c_{k\sigma}^+(0) \rangle = - \int_{-\infty}^{\infty} d\omega \left[\frac{e^{-\omega\tau}}{1 + e^{-\beta\omega}} \right] A(k, \omega)$$

Extract : Density of States Frequency dependent Pair Susceptibility

MEM extensive checks : sum rules, moments, Kramers-Kronig

Superfluid density

T_c

Current-current correlations

$$\Lambda_{xx}(l, \tau) = \langle j_x(l, \tau) j_x(0, 0) \rangle$$

$$j_x(\mathbf{l}\tau) = e^{H\tau} \left(i t \sum_{\sigma} (c_{\mathbf{l}+\hat{x}, \sigma}^{\dagger} c_{\mathbf{l}, \sigma} - c_{\mathbf{l}, \sigma}^{\dagger} c_{\mathbf{l}+\hat{x}, \sigma}) \right) e^{-H\tau}$$

$$\Lambda_{xx}(\mathbf{q}; \tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \frac{\exp(-\omega\tau)}{[1 - \exp(-\beta\omega)]} \text{Im} \Lambda_{xx}(\mathbf{q}; \omega)$$

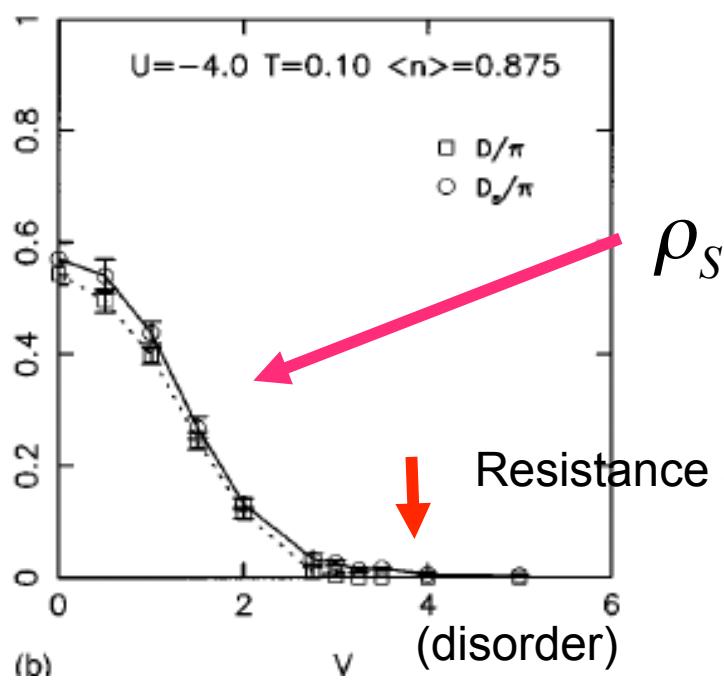
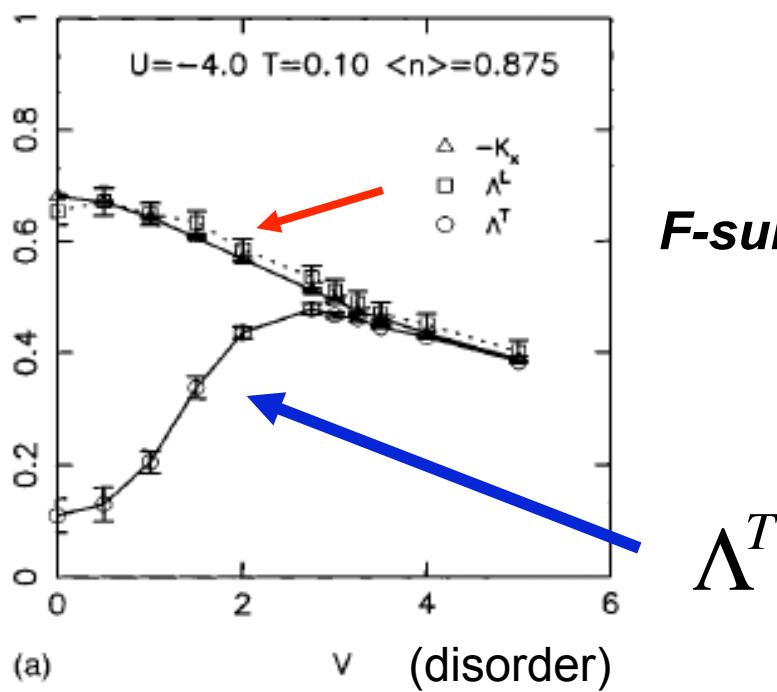
$$\Lambda^L \equiv \lim_{q_x \rightarrow 0} \Lambda_{xx}(q_x, q_y = 0, \omega = 0) = -K_x \quad \rightarrow \rho$$

$$\Lambda^T \equiv \lim_{q_y \rightarrow 0} \Lambda_{xx}(q_x = 0, q_y, \omega = 0) \quad \rightarrow \rho_n$$

$$\rho_S = \Lambda^L - \Lambda^T$$

Superfluid density

F-sum rule



$$\Lambda^L \equiv \lim_{q_x \rightarrow 0} \Lambda_{xx}(q_x, q_y = 0, \omega = 0) = -K_x$$

$$\Lambda^T \equiv \lim_{q_y \rightarrow 0} \Lambda_{xx}(q_x = 0, q_y, \omega = 0)$$

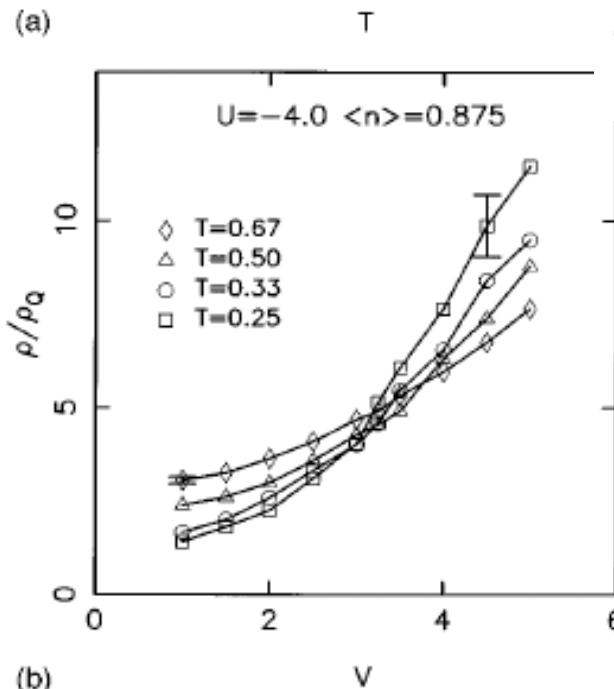
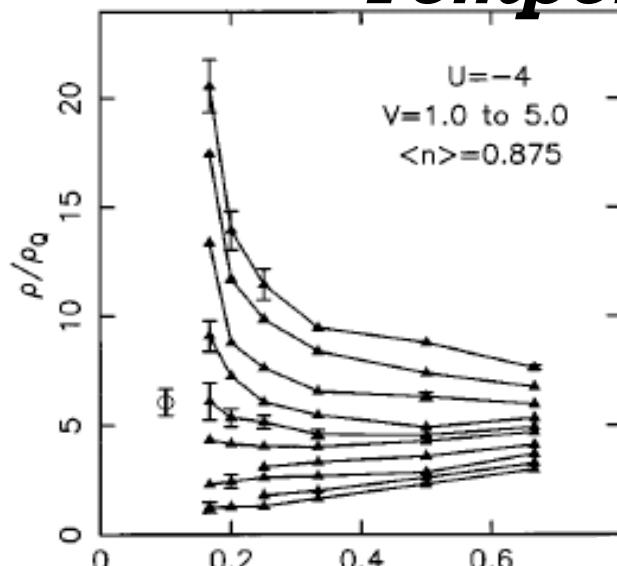
$$\rho_S = \Lambda^L - \Lambda^T$$

$$\Lambda^T$$

$$\rho_S$$

Resistance at the transition \sim quantum of resistance

Temperature-dependent resistivity



$$\text{Re } \sigma(\omega) = D\delta(\omega) + \sigma_{reg}(\omega)$$

$$\text{Re } \sigma_{reg}(\omega) = \text{Im } \Lambda(\omega) / \omega$$

$$\Lambda_{xx}(\mathbf{q}; \tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \frac{\exp(-\omega \tau)}{[1 - \exp(-\beta\omega)]} \text{Im} \Lambda_{xx}(\mathbf{q}; \omega),$$

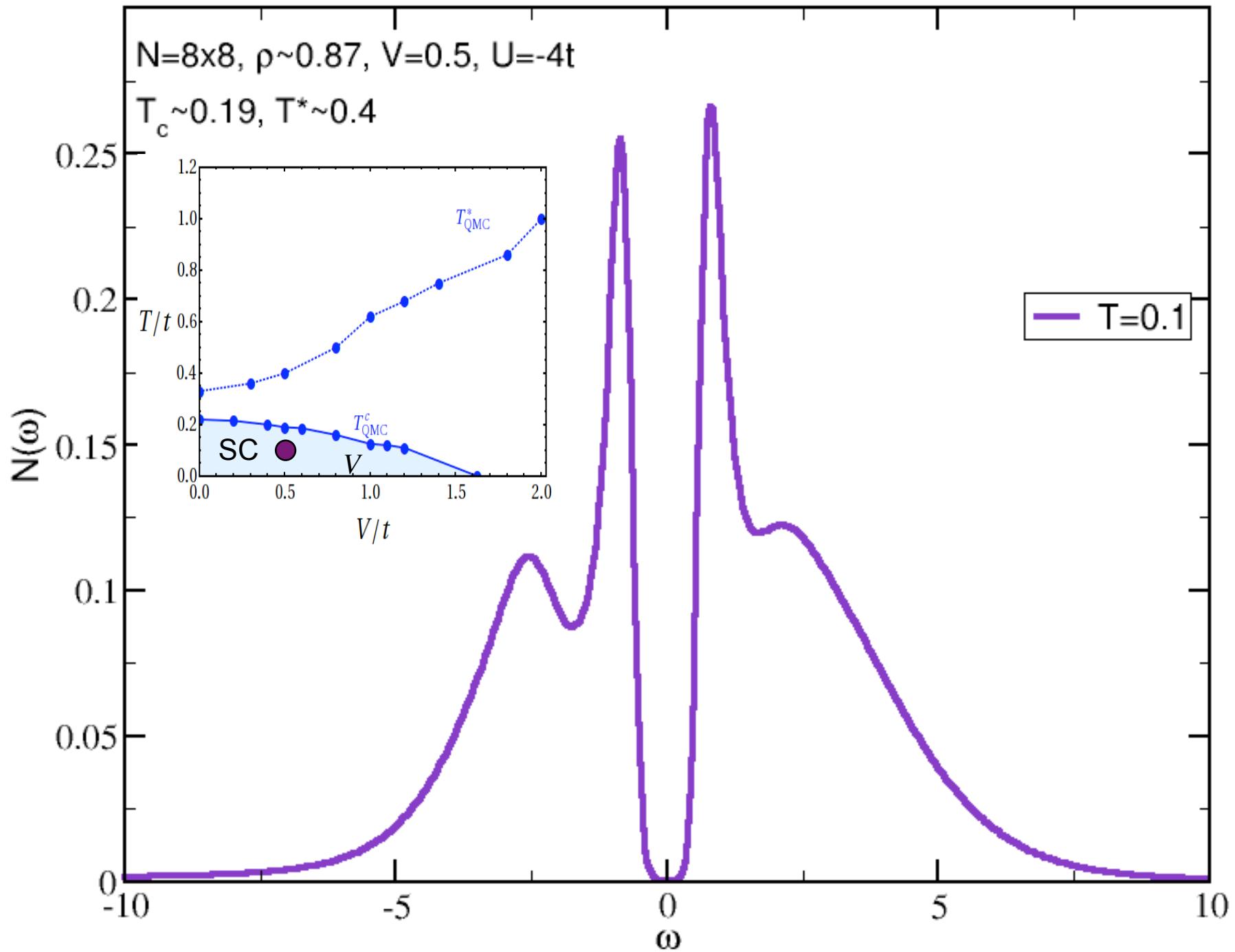
$$\Lambda_{xx}(\mathbf{q}=0; \tau=\beta/2) = \pi \sigma_{dc} / \beta^2$$

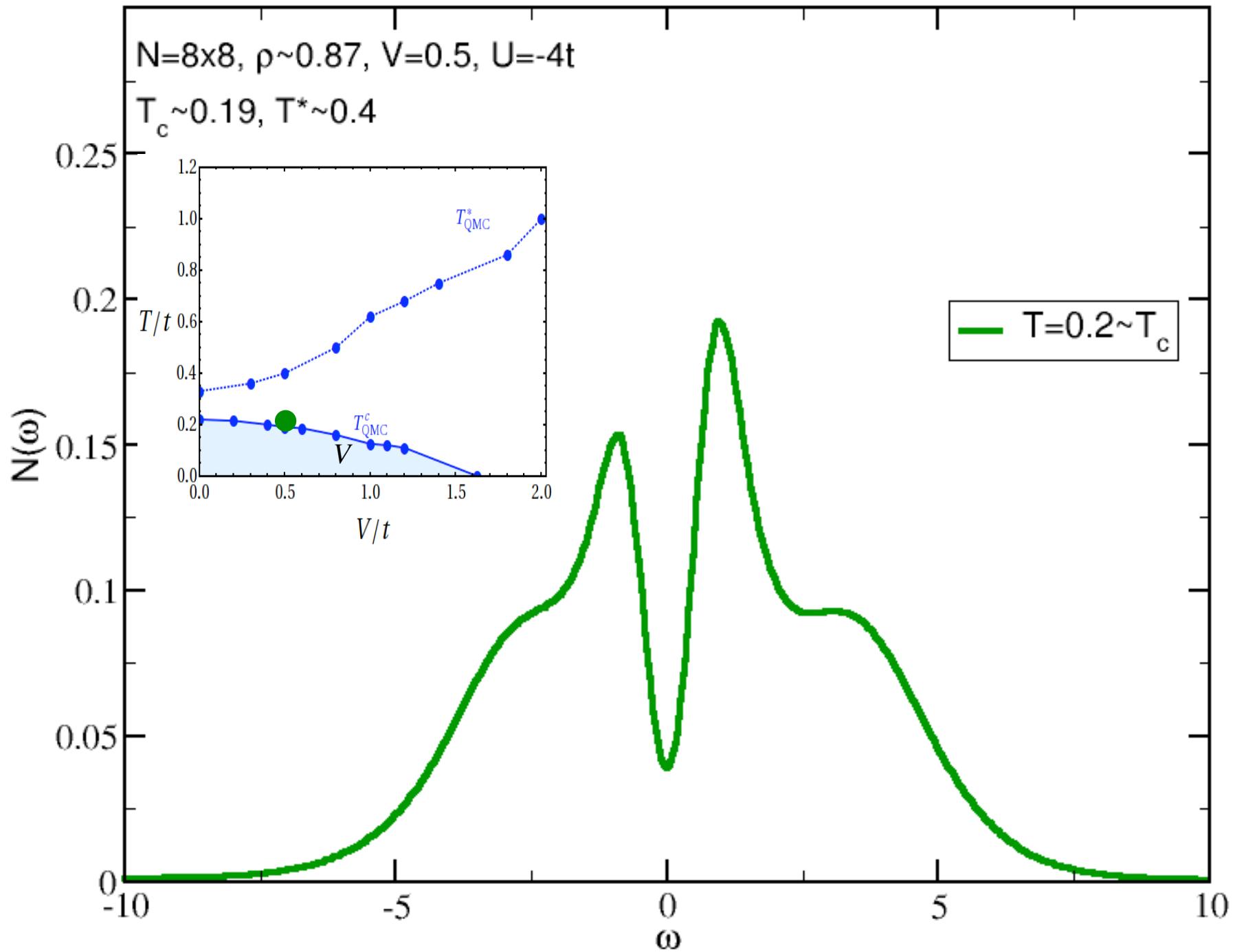


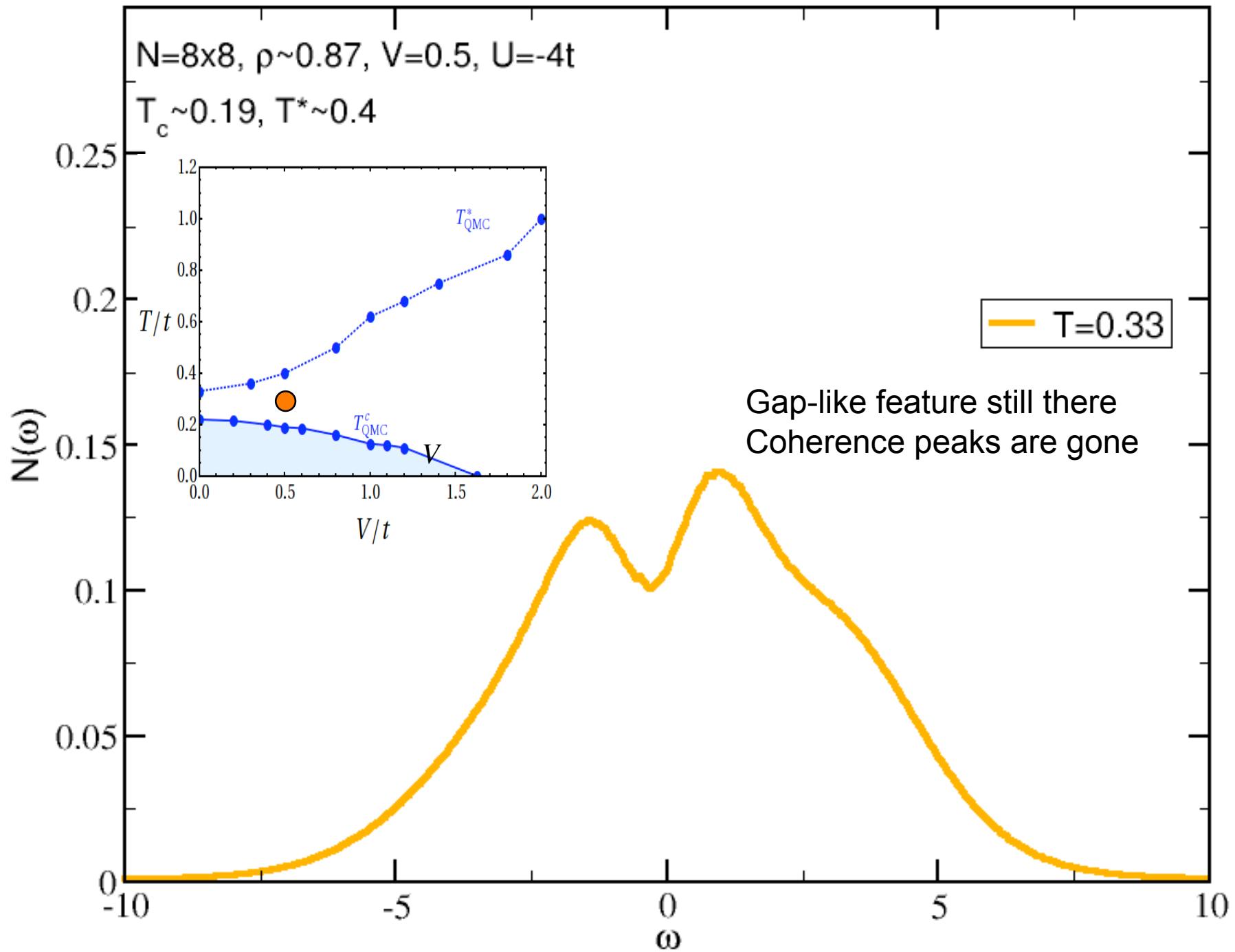
dc resistivity

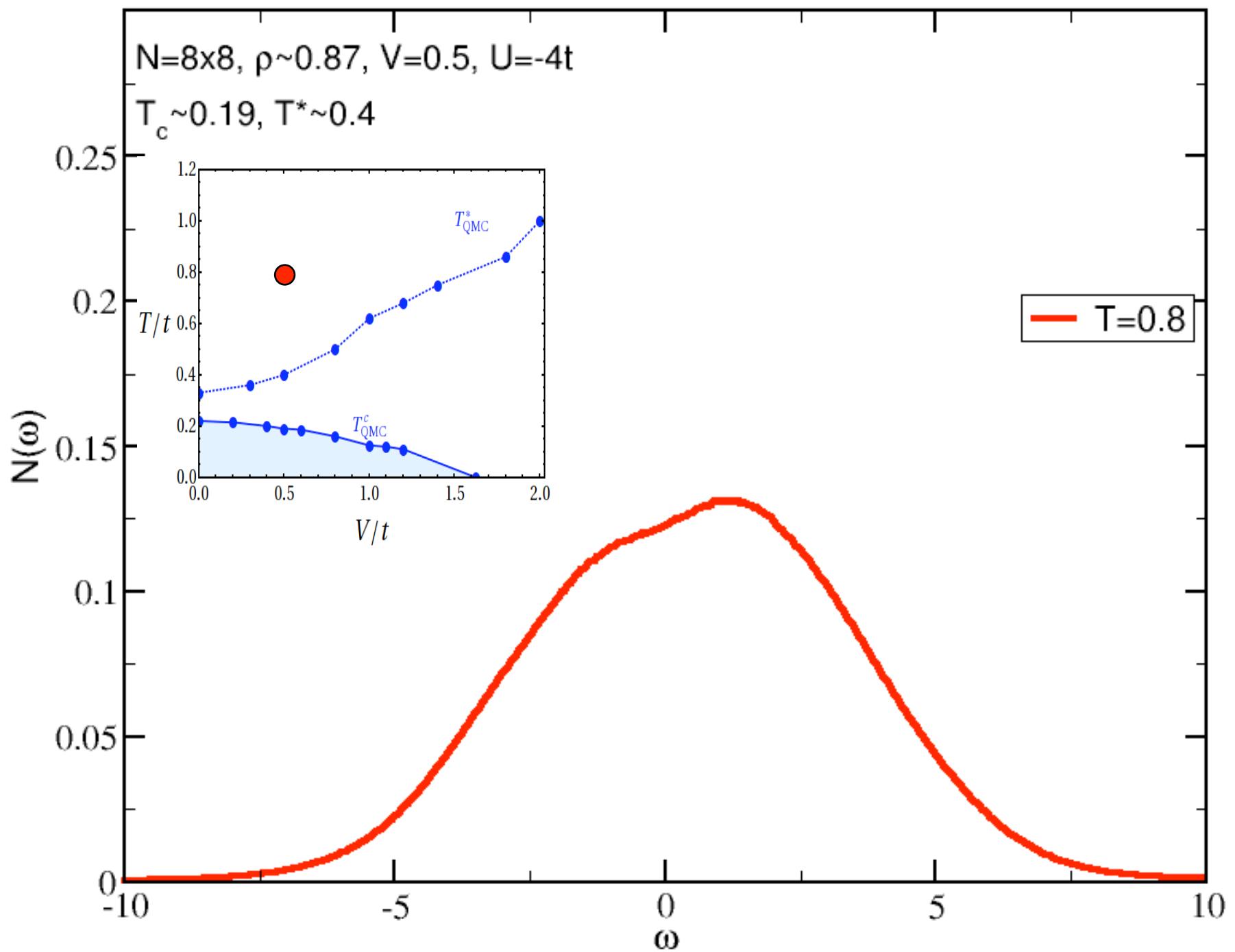
NT, Scaletter & Randeria PRB 54, 3756 (1996)

Single Particle Spectral properties

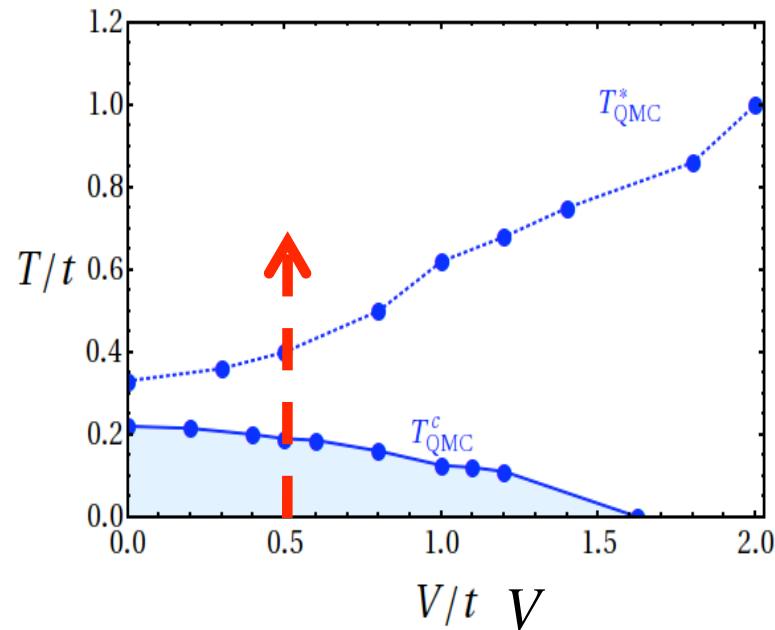








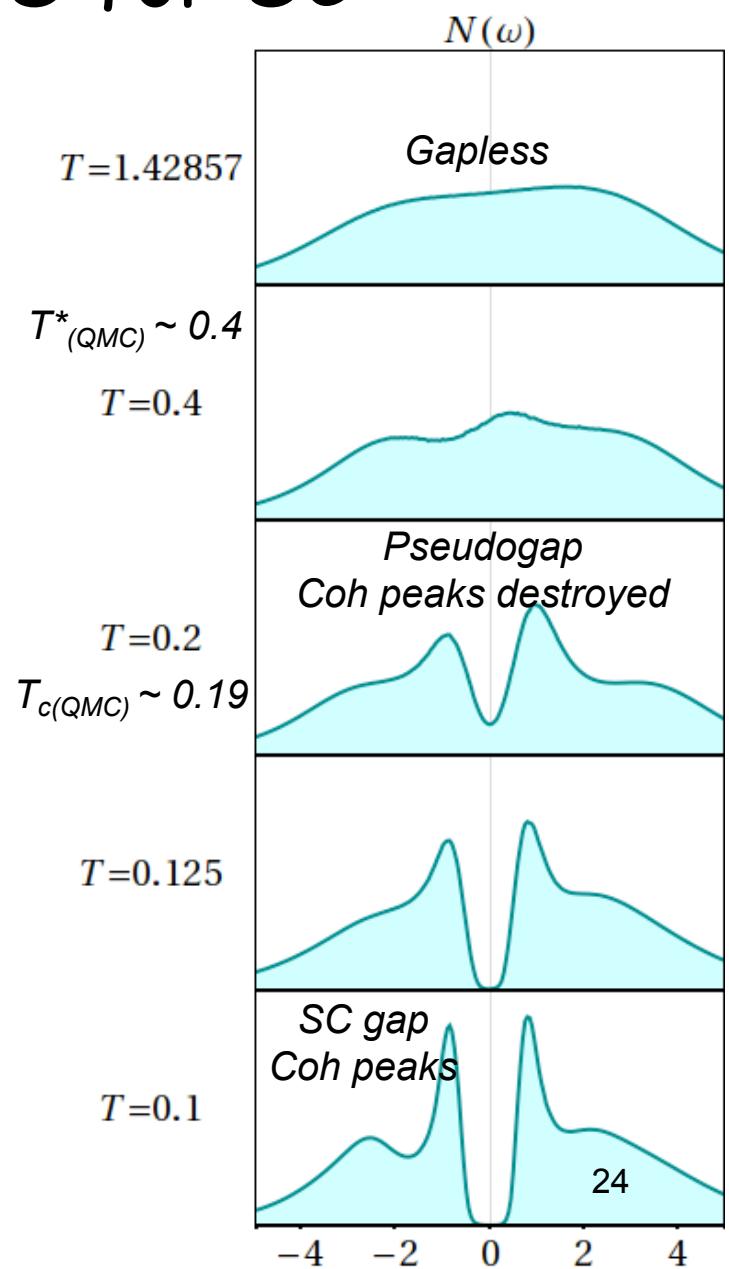
T dependence of DOS for SC



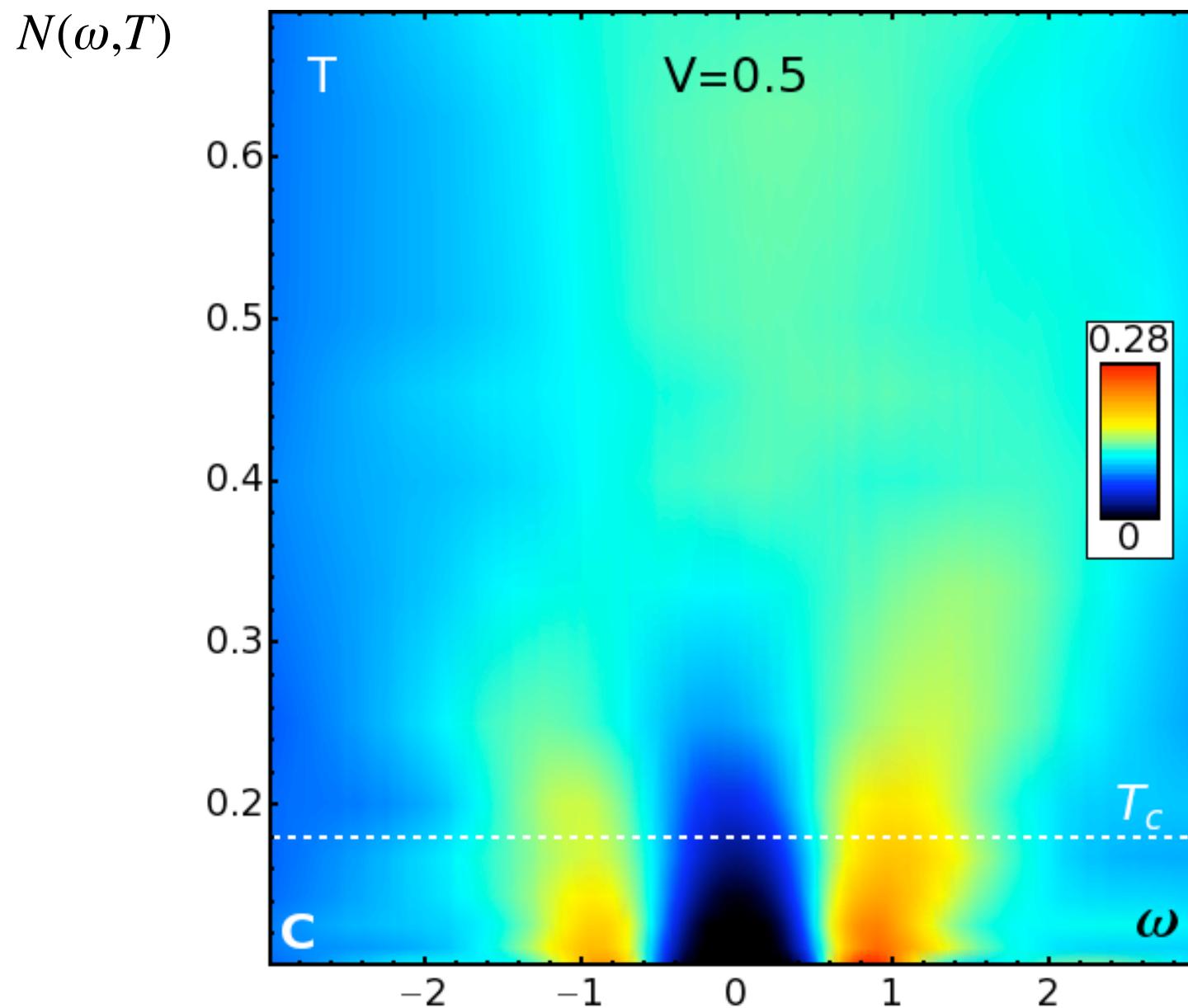
PSEUDOOGAP OPENED BY DISORDER

** Have to go beyond BdG to capture the effects of thermal phase fluctuations

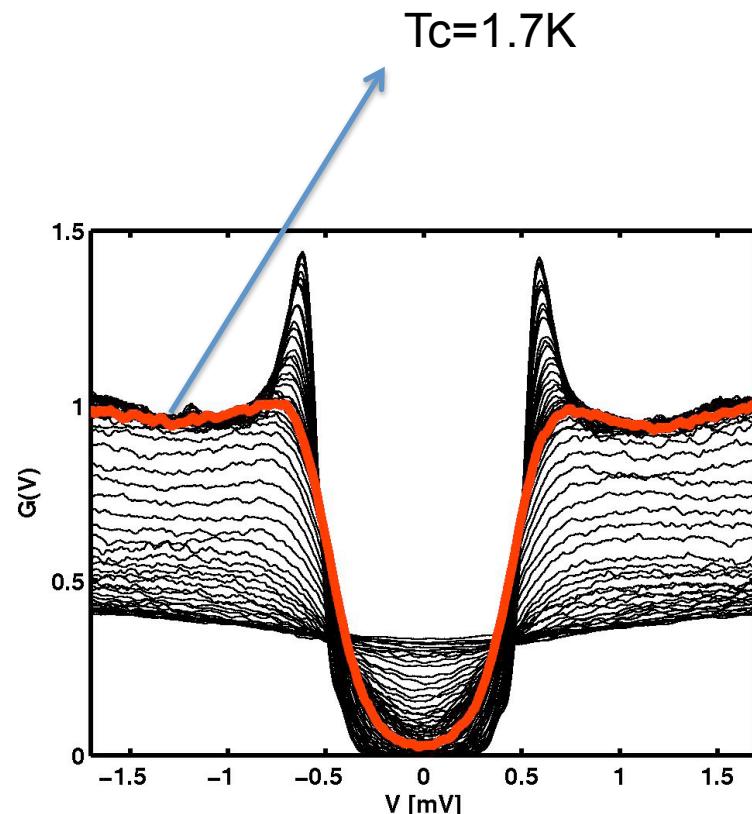
$V=0.5$



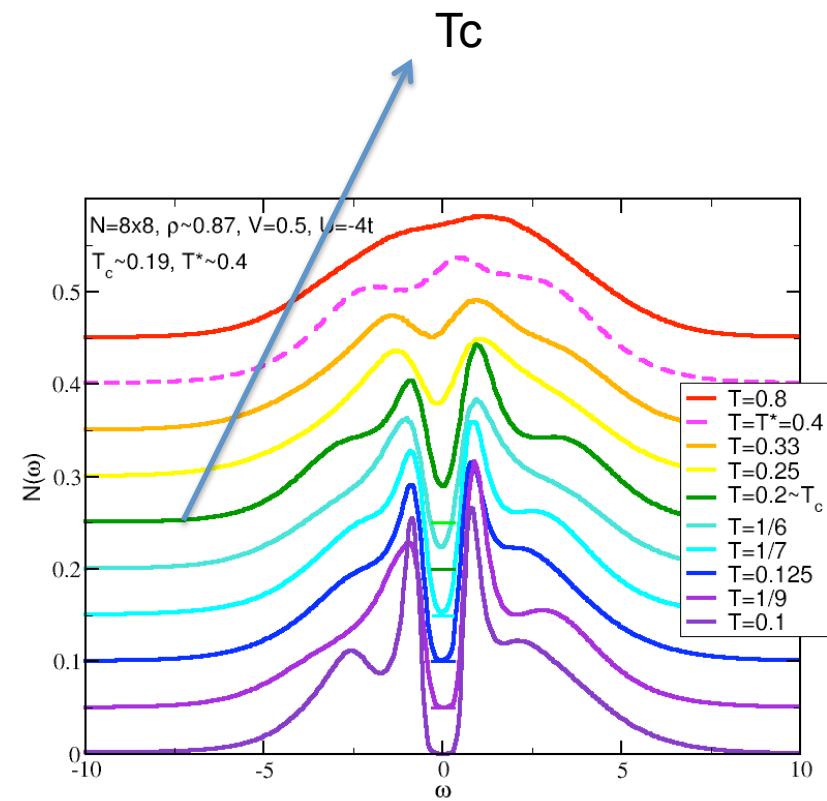
Single Particle Density of States



Temperature Dependence of DOS: Comparison with experiments

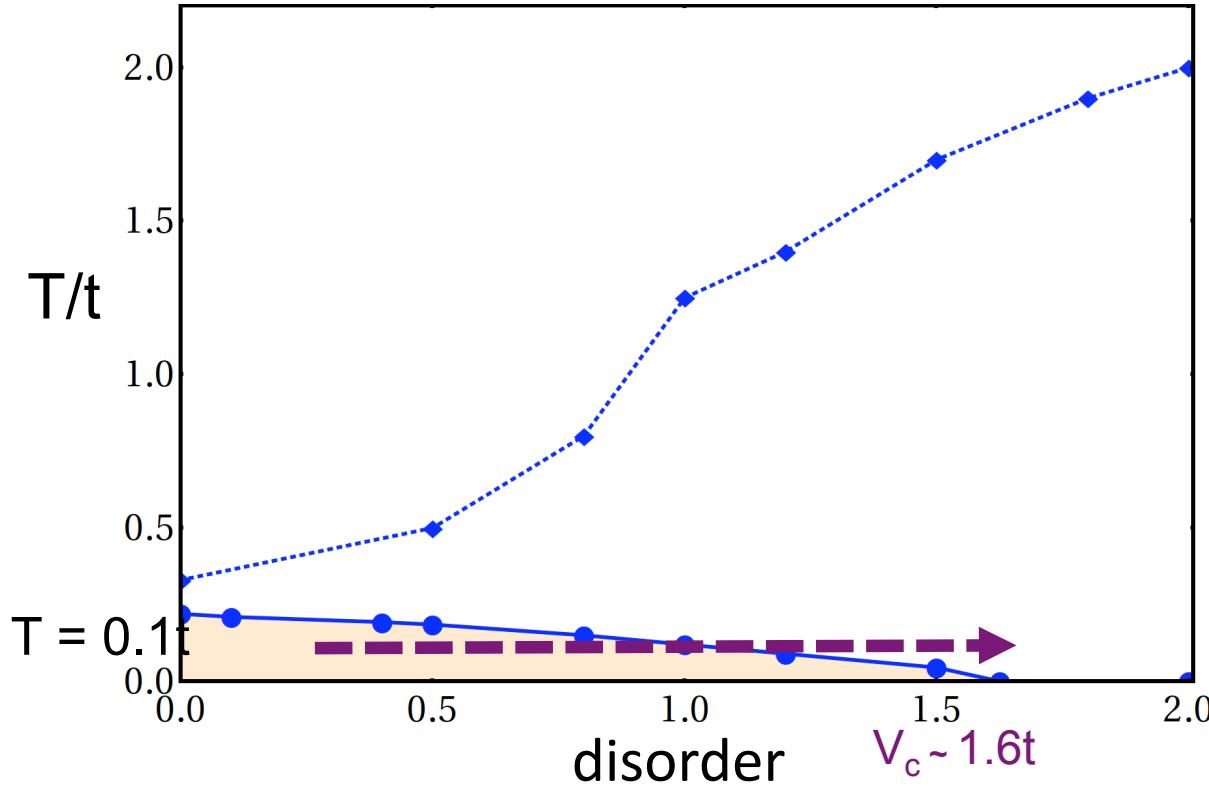


Experiments:
Scanning tunneling spectroscopy on InOx
(B. Sacépé et al.)



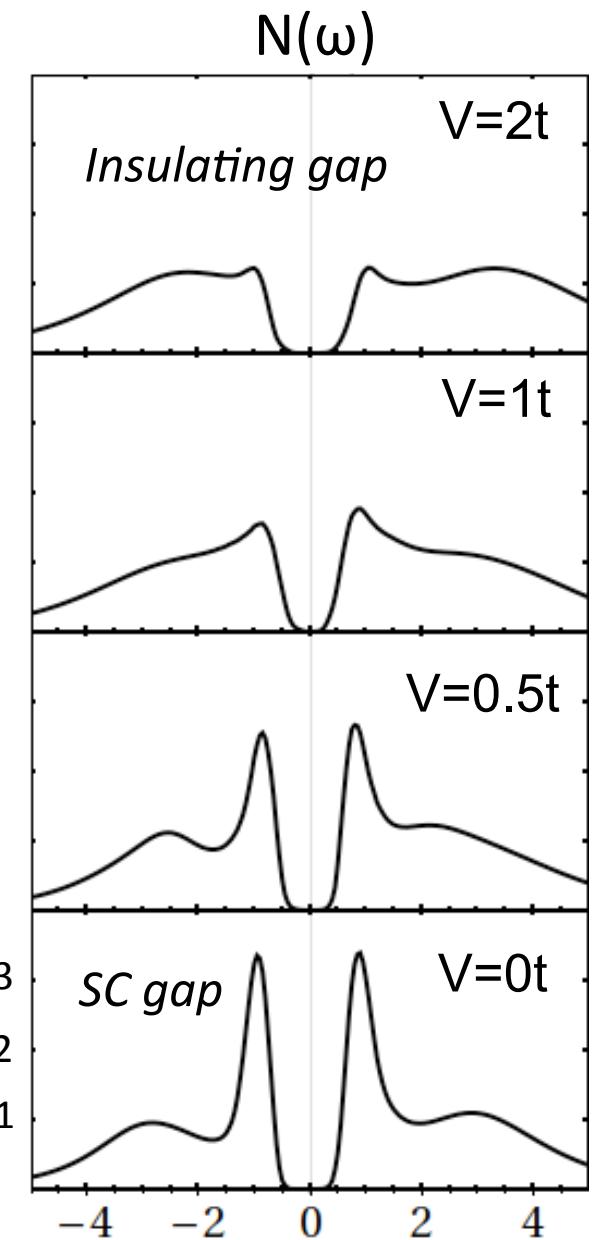
Theory:
determinant quantum Monte Carlo

Effect of quantum fluctuations at low T

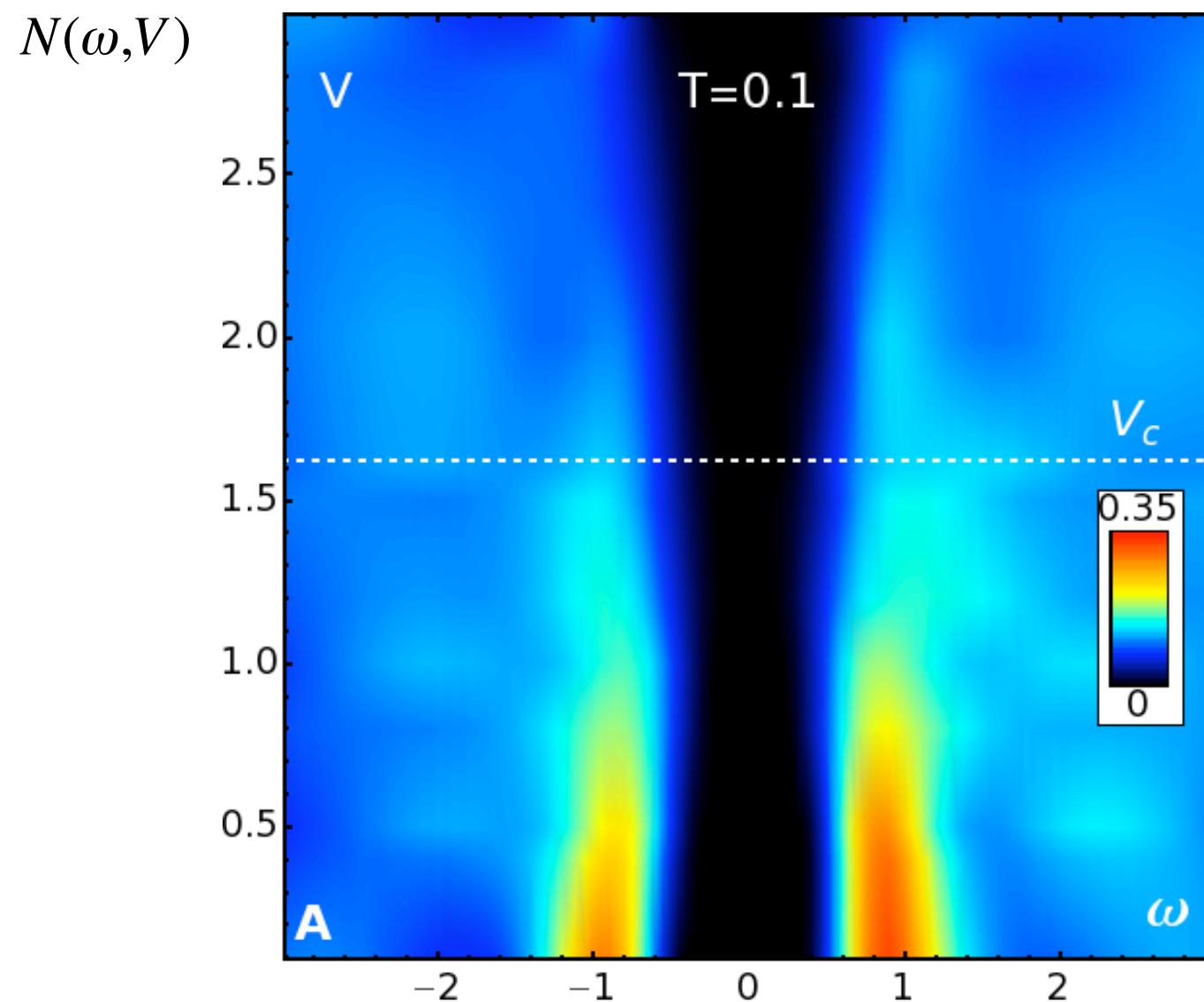


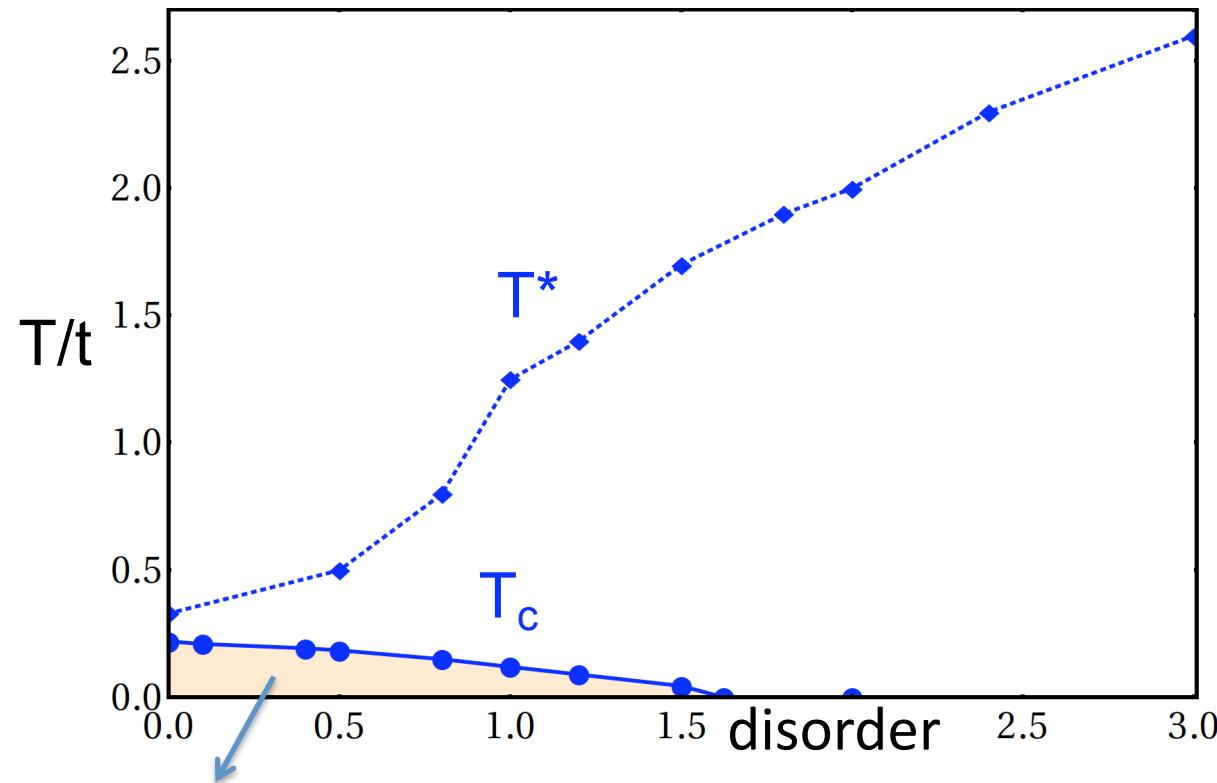
- Gap is **robust** against disorder
- Gap increases at very large disorder
- **Loss of coherence peaks** at $V_c \sim 1.6t$

V_c from superfluid stiffness



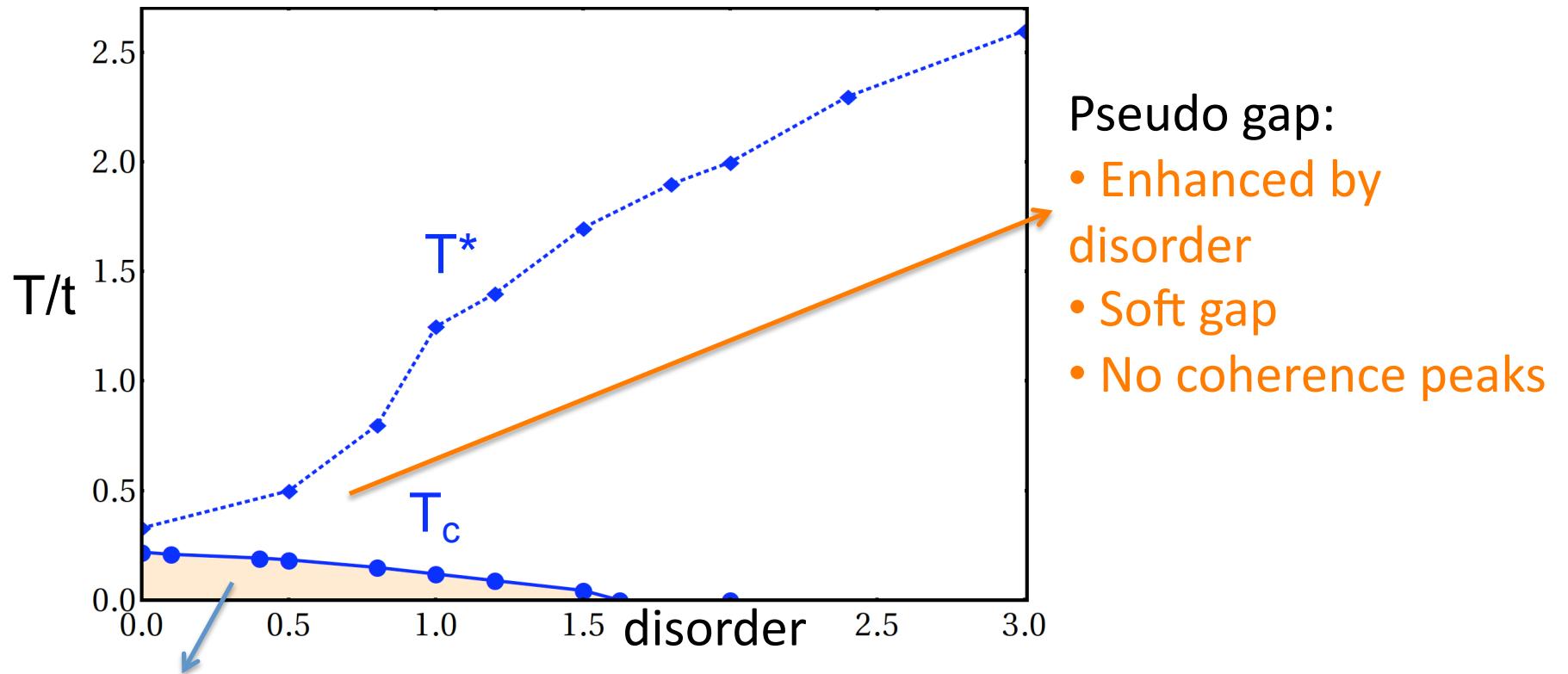
Single Particle Density of States





Superconductor

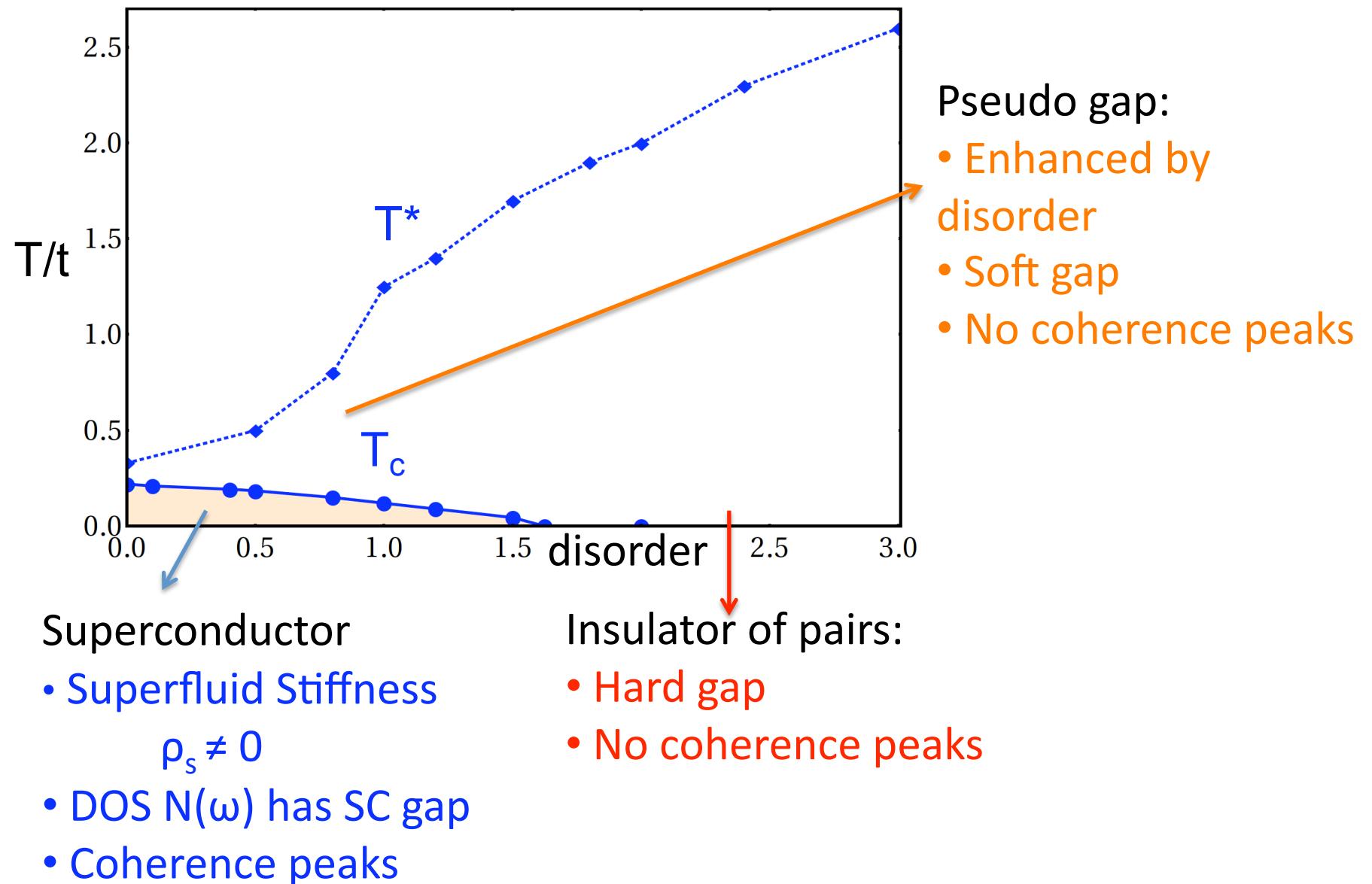
- Superfluid Stiffness
 $\rho_s \neq 0$
- DOS $N(\omega)$ has SC gap
- *Coherence peaks*



Superconductor

- Superfluid Stiffness
 $\rho_s \neq 0$
- DOS $N(\omega)$ has SC gap
- Coherence peaks

Main Results :

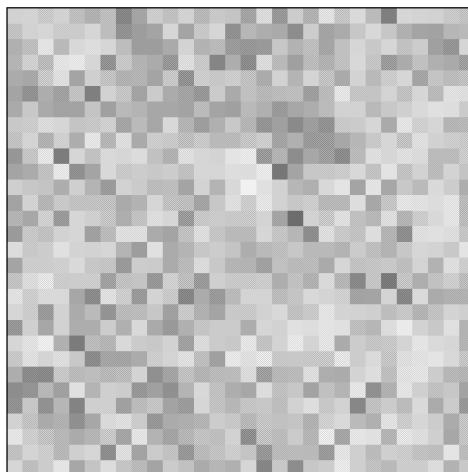


Why is the gap finite?

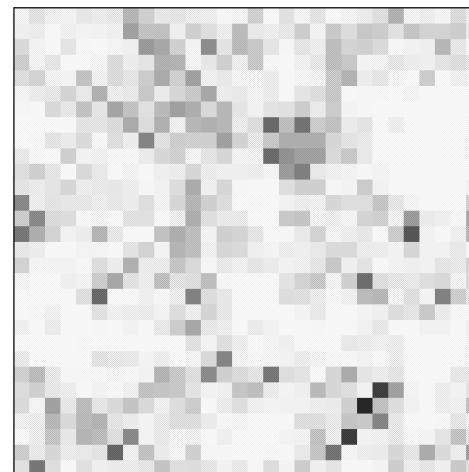


Spatial map of pairing amplitude

$$\Delta(r) = \langle c_{r\uparrow}^+ c_{r\downarrow}^+ \rangle$$



$V = t$



$V = 2t$

Given realization of disorder

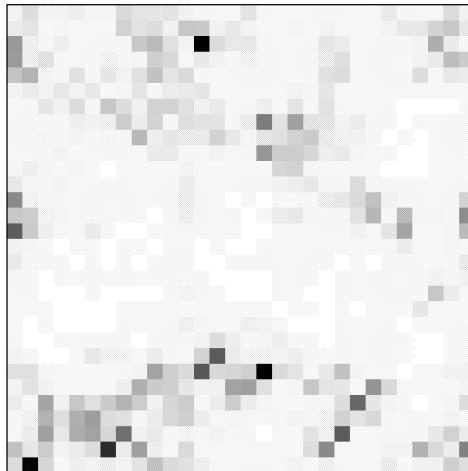
$V(r)$ varies on scale of a

$$U = 1.5t$$

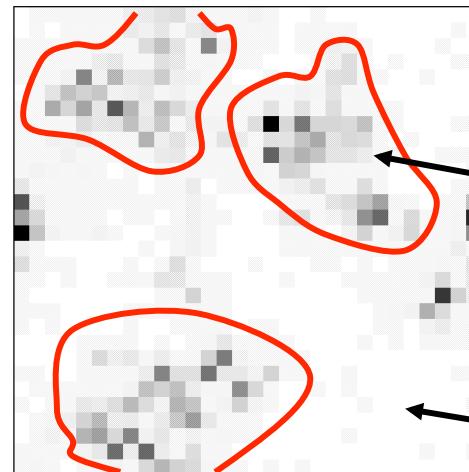
$$\langle n \rangle = 0.875$$

$$\xi \approx 10a$$

$$N = 30 \times 30$$



$V = 2.5t$



$V = 3t$

Structures generated on scale ξ

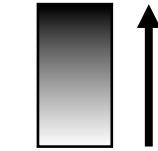
Formation of SC blobs

Δ large

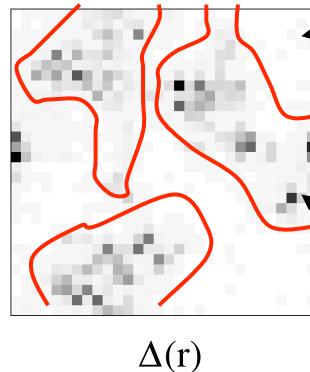
Insulating Sea

$\Delta \approx 0$

Why is the gap finite? Where do excitations live?



Pairing
amplitude
map $\Delta(r)$
(a)

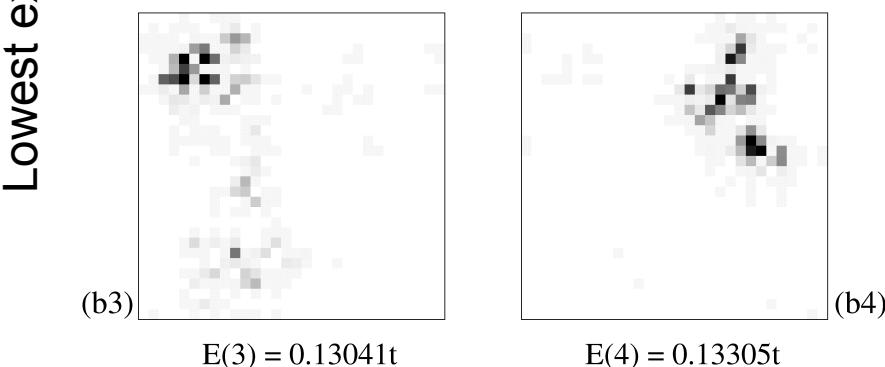
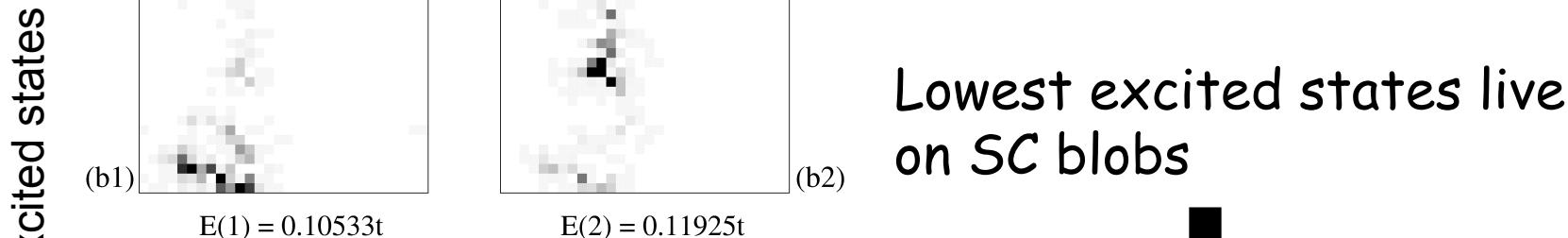


$\Delta \sim 0$

high hills: empty

deep valleys: trapped pairs
no number fluctuations

SC islands formed where
 $|V(r) - \mu|$ is small



Ghosal, Randeria, Trivedi PRL 81, 3940 (1998);
PRB 65, 14501 (2002)

INHOMOGENOUS PAIRING OF EXACT EIGENSTATES

$$|U| \ll t \ll V$$

↓

Find the exact eigenstates of non-interacting problem with disorder

$$H_0 |\phi_\alpha\rangle = \varepsilon_\alpha |\phi_\alpha\rangle$$

Next pair time reversed states $(\phi_\alpha, \phi_{\bar{\alpha}})$

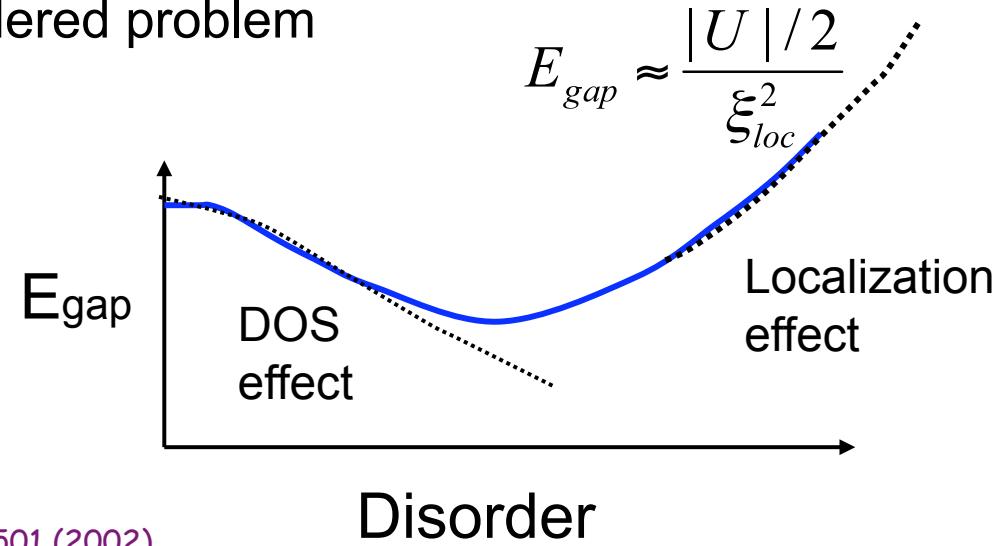
Gap equation

$$\Delta_\alpha = \frac{|U|}{2} \sum_\beta M_{\alpha\beta} \frac{\Delta_\beta}{(\xi_\beta^2 + \Delta_\beta^2)^{1/2}}$$

$$M_{\alpha\beta} = \sum_r |\phi_\alpha(r)|^2 |\phi_\beta(r)|^2$$

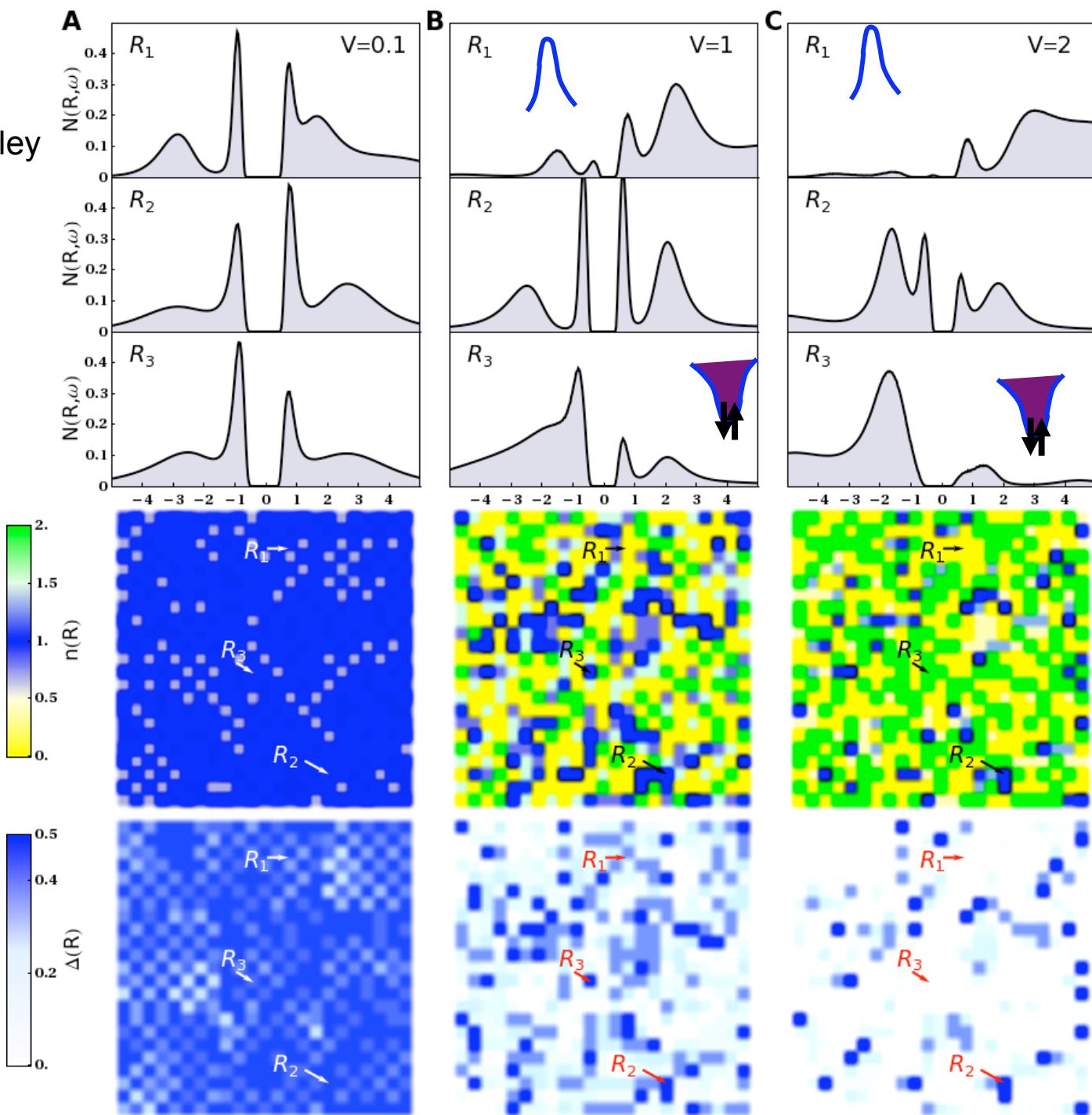
determined by properties of non-interacting disordered problem

$$= \begin{cases} 1/N & (\text{Low } V) \\ \delta_{\alpha\beta} \sum_r |\phi_\alpha(r)|^4 \approx \frac{1}{\xi_{loc}^2(\alpha)} & (\text{high } V) \end{cases}$$

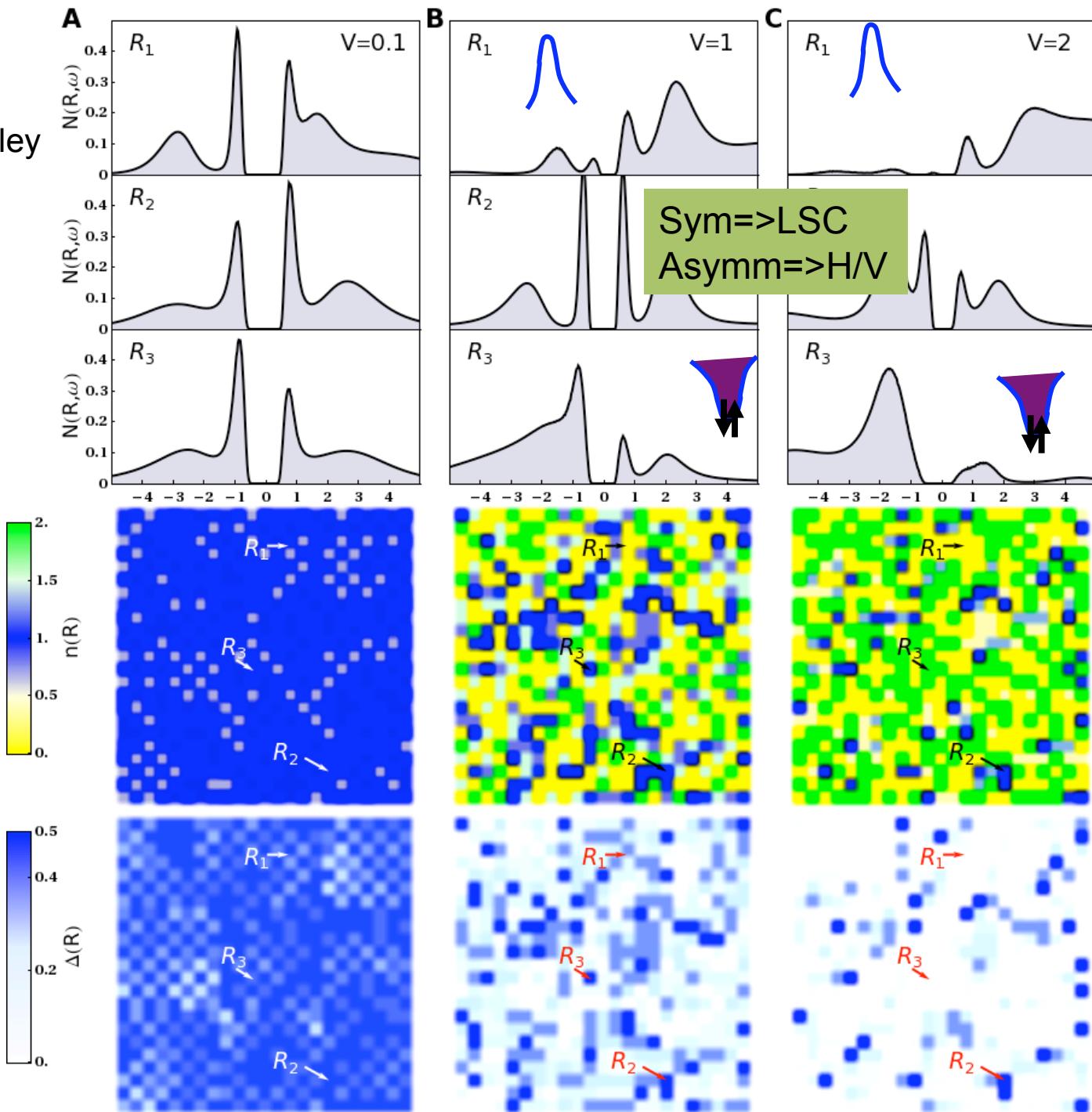


Ghosal, Randeria, Trivedi PRL 81, 3940 (1998); PRB 65, 14501 (2002)
Feigelman et. al PRL 98, 027001 (2007)

R1=high hill
 R3=deep valley
 R2=plateau



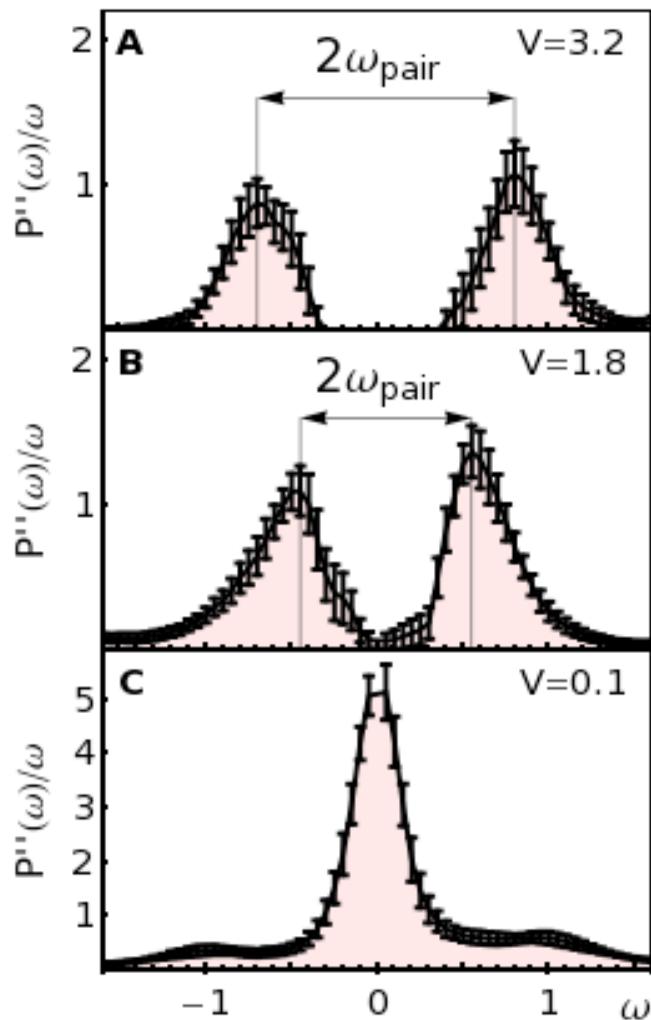
R1=high hill
 R3=deep valley
 R2=plateau



Two-particle spectral properties

Pair Susceptibility

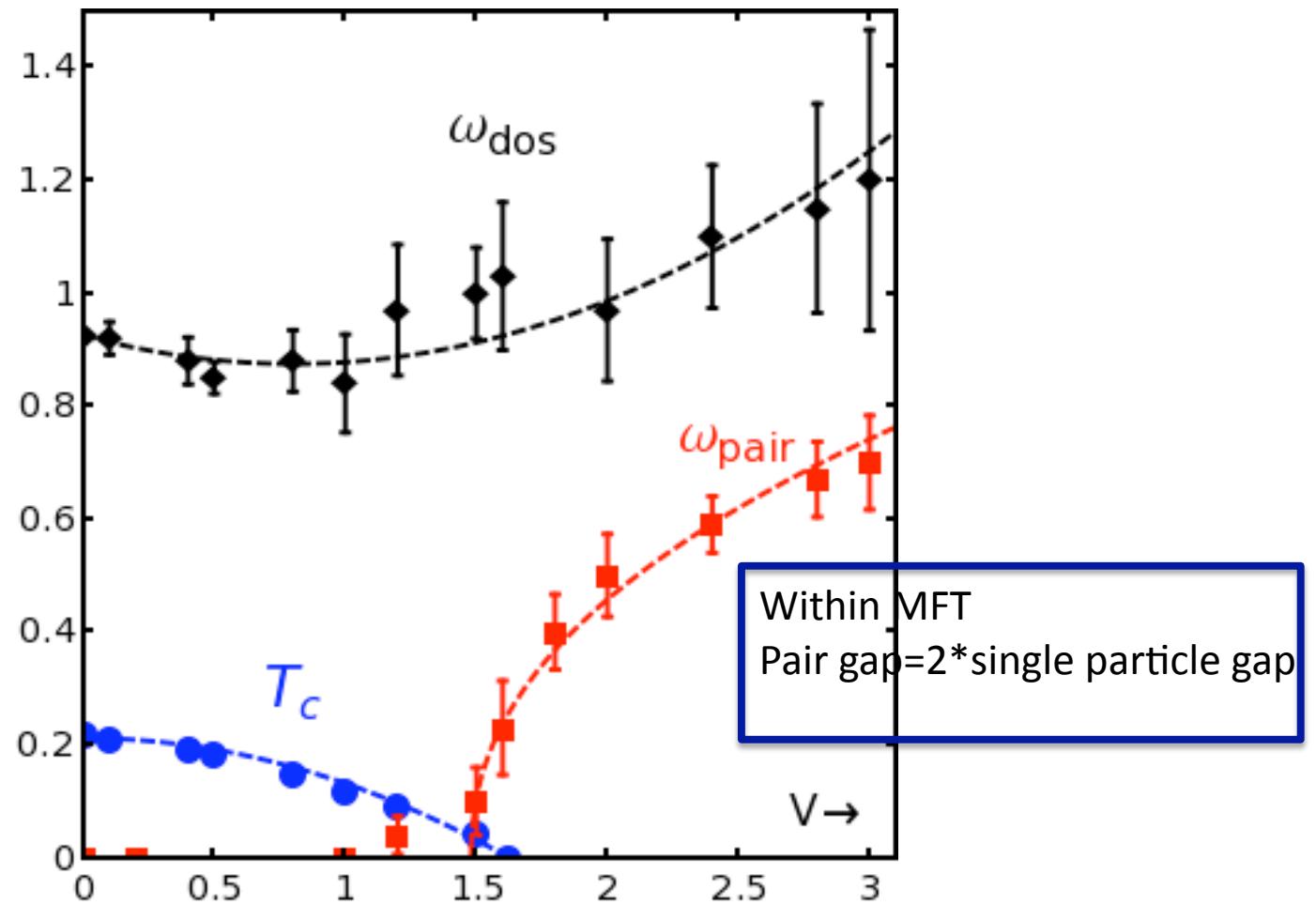
Amplitude for a pair injected at site R at time t=0
to be found at the same site at a later time tau



$$P(\tau) = \sum_R \langle T_\tau F(R, \tau) F^+(R, 0) \rangle = \int_{-\infty}^{\infty} d\omega \left[\frac{e^{-\tau\omega}}{e^{-\beta\omega} - 1} \right] \frac{(-1)P''(\omega)}{\pi}$$

$$F(R, \tau) = c_{R\downarrow}(\tau) c_{R\uparrow}(\tau)$$

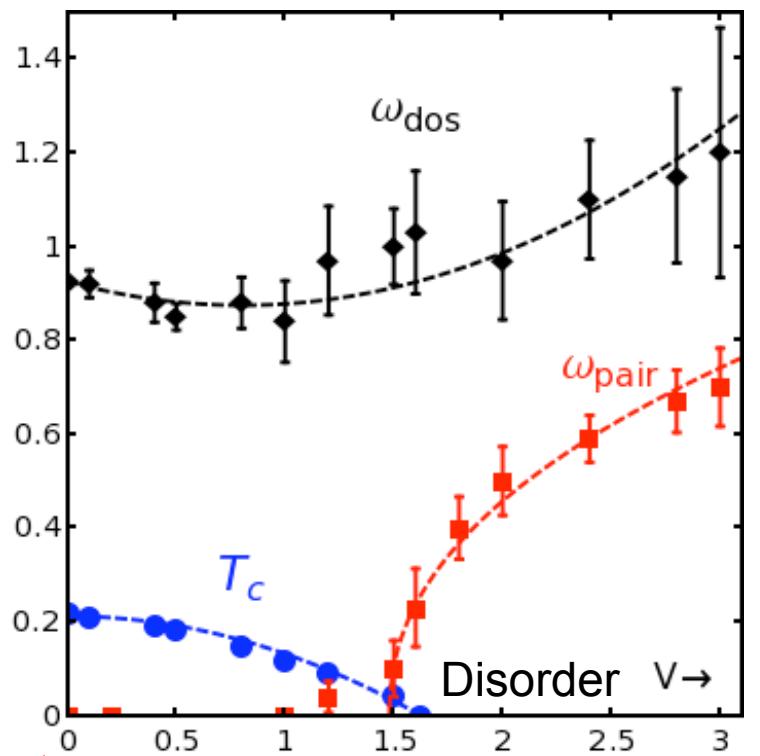
Tiny contribution from rare regions at low energies
Focus on dominant two-peak structure in insulator



Phase Diagram SC-I Transition driven by disorder

Superconductor

- Superfluid Stiffness $\rho_s \neq 0$
- DOS $N(\omega)$ has SC gap
- Coherence peaks

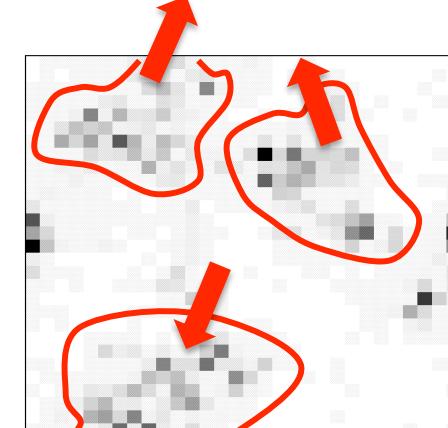
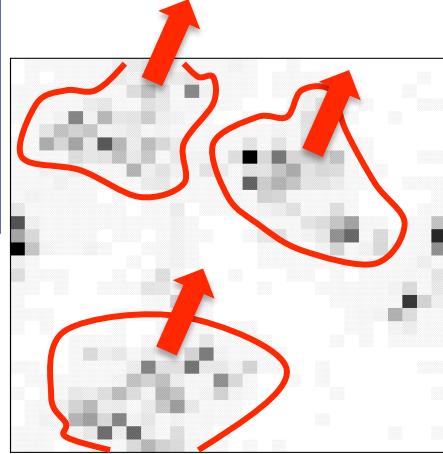


Pseudo gap:

- Enhanced by disorder
- Soft gap
- No coherence peaks

Insulator of pairs:

- Hard gap
- No coherence peaks
- ω_{pair} from gap scale in pair susceptibility

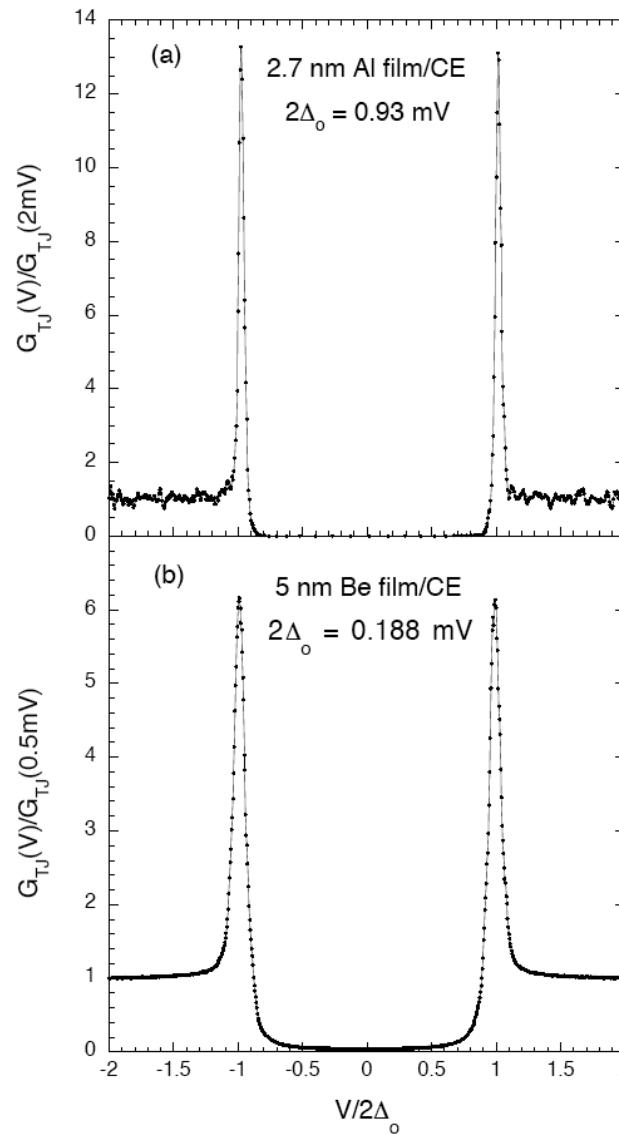


Part II:

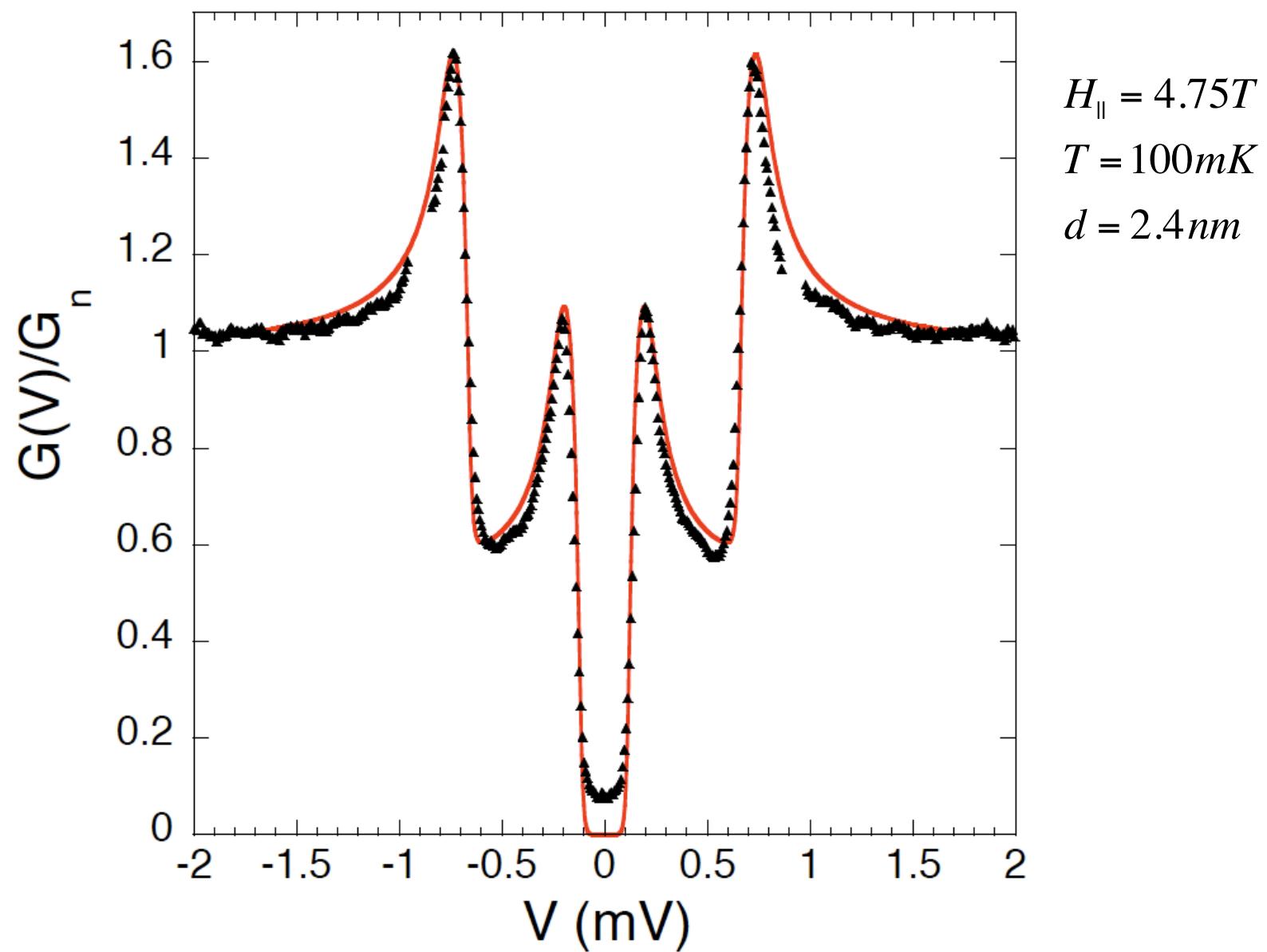
Zeeman Field Driven SC-N transition

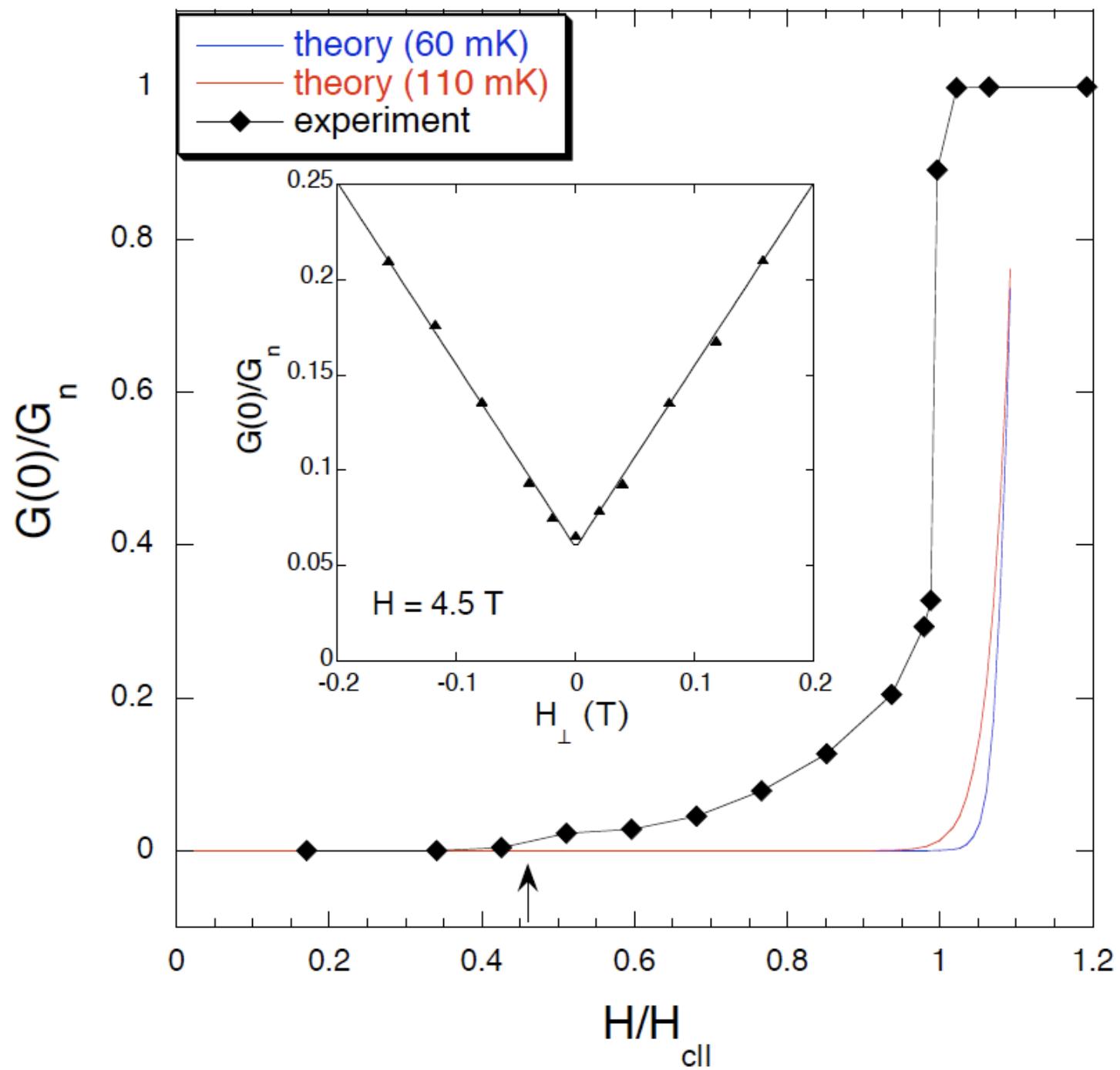
Y. L. Loh, N. Trivedi, Y. M. Xiong, P. W. Adams, and G. Catelani
Mystery of Excess Low Energy States
in a Disordered Superconductor in a Zeeman Field, arXiv

Tunneling DOS in Zero Field

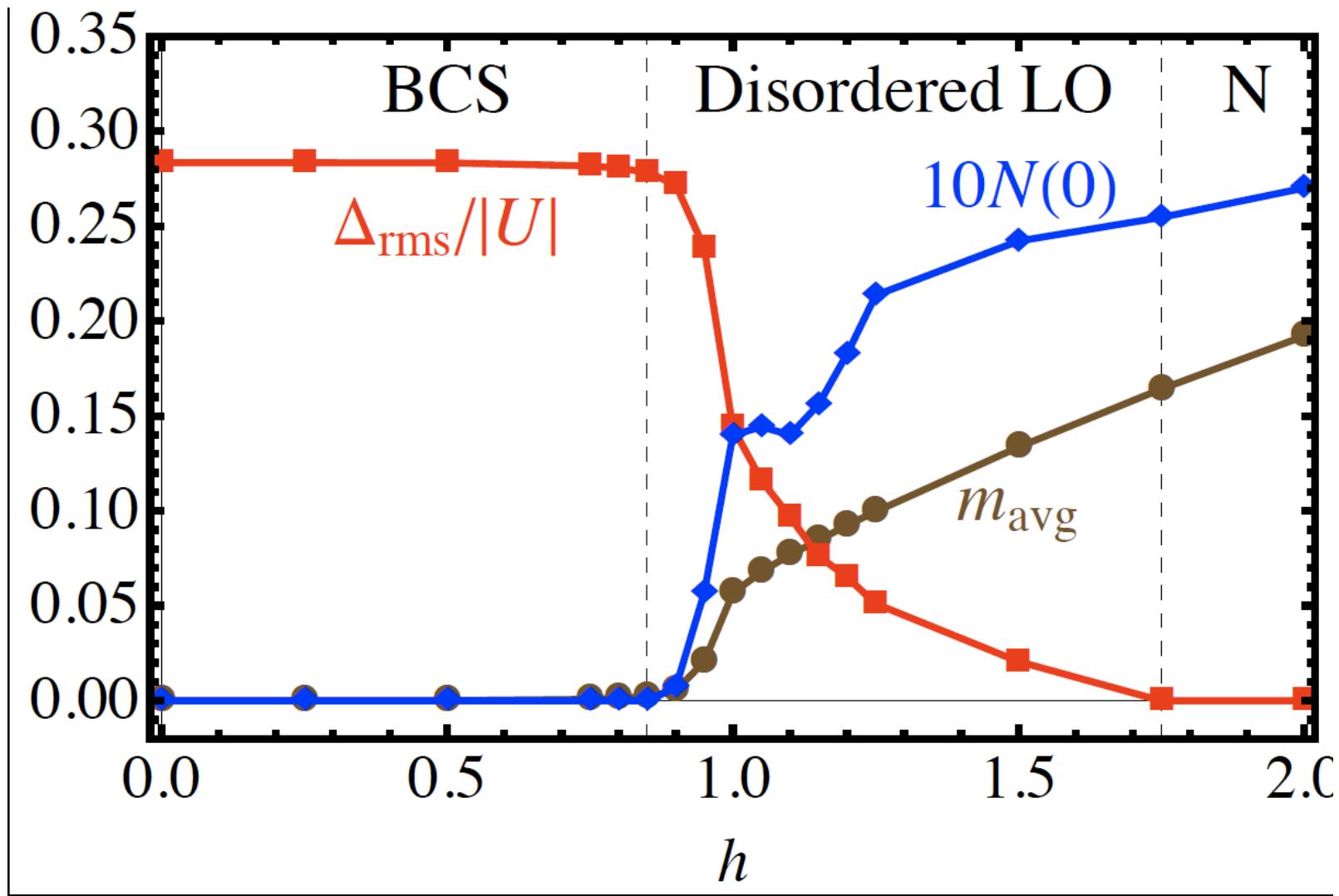


T = 80 mK

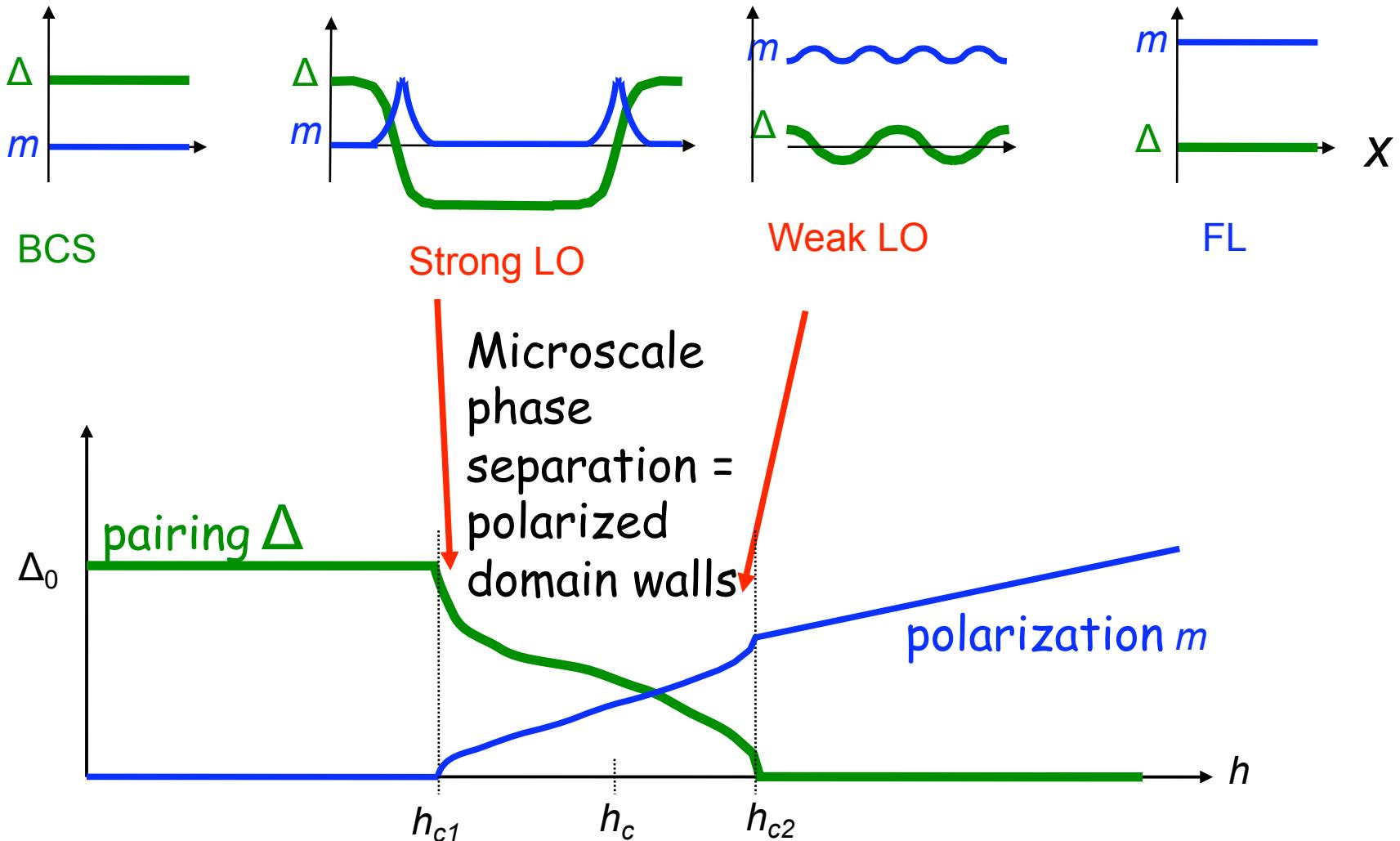




Where are these excess low energy states coming from??



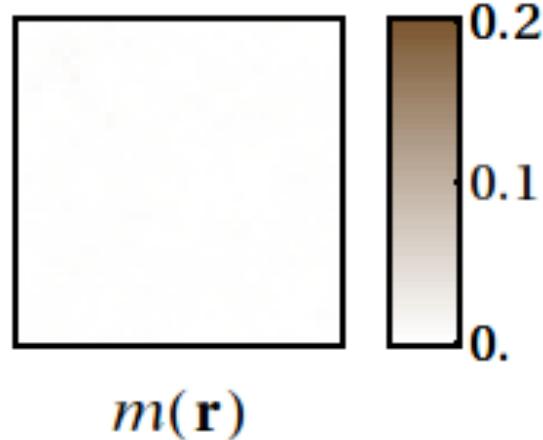
Modulated (LO) SC order parameter



$V = 2t$

$U = -4t$

Local magnetization

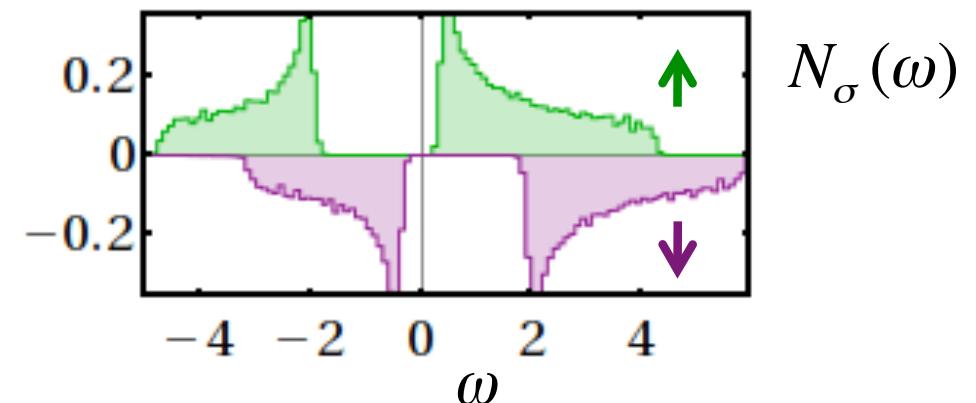


$m(\mathbf{r})$

Disorder + Zeeman field

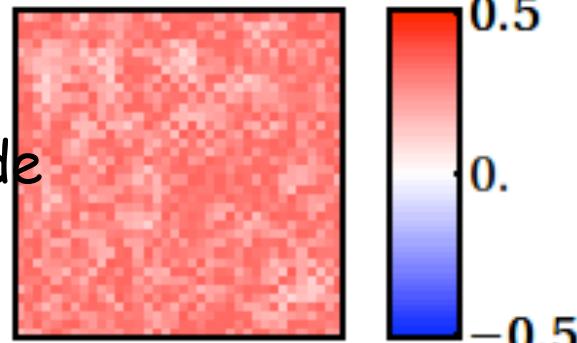
$h = 0.8$

Spin resolved DOS

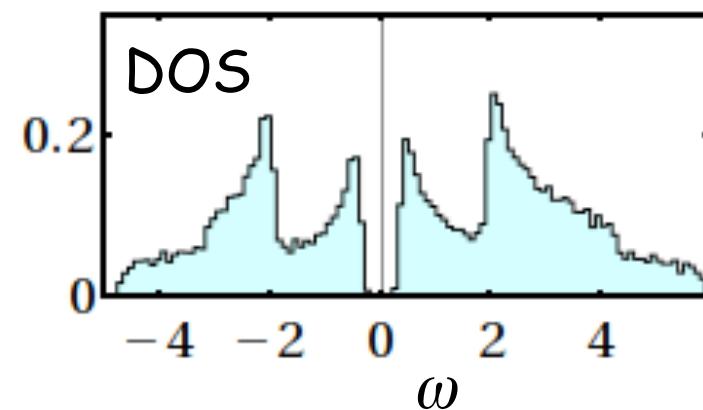


$N_\sigma(\omega)$

Local
Pairing
amplitude



$F(\mathbf{r})$



$N(\omega)$

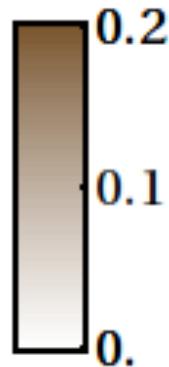
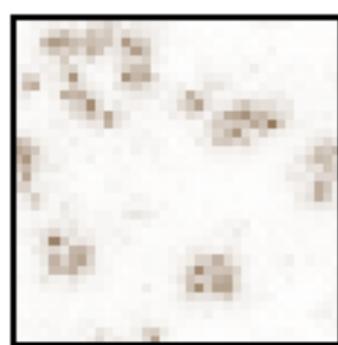
I. Paired unpolarized SC

Disorder + Zeeman field

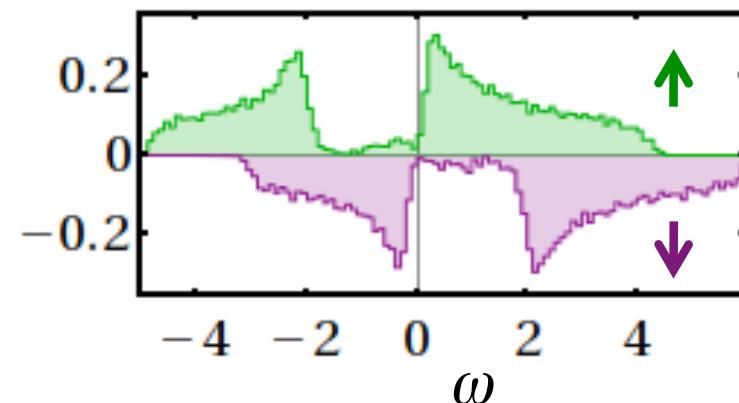
$V = 2t$

$h = 0.95$

$U = -4t$

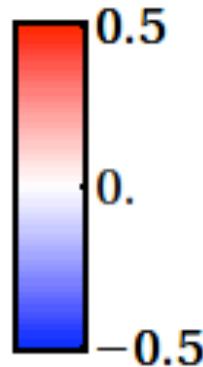
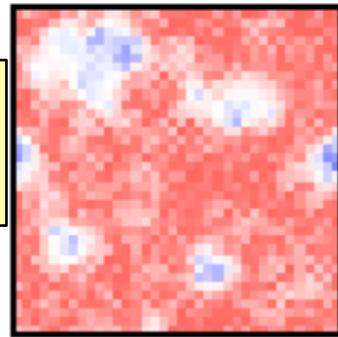


$m(\mathbf{r})$

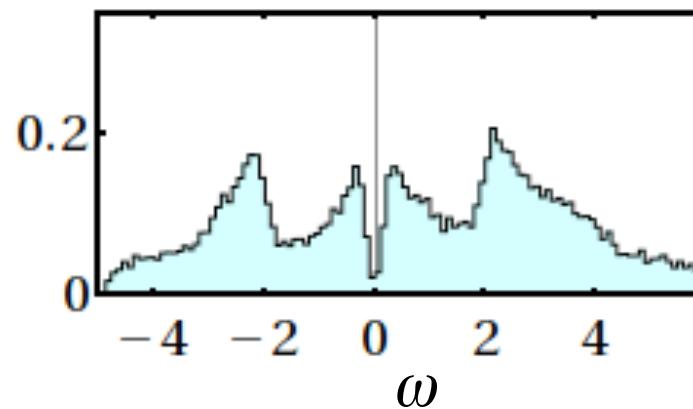


$N_\sigma(\omega)$

+ and - domains



$F(\mathbf{r})$



$N(\omega)$

Disordered LO

soft gap

End....